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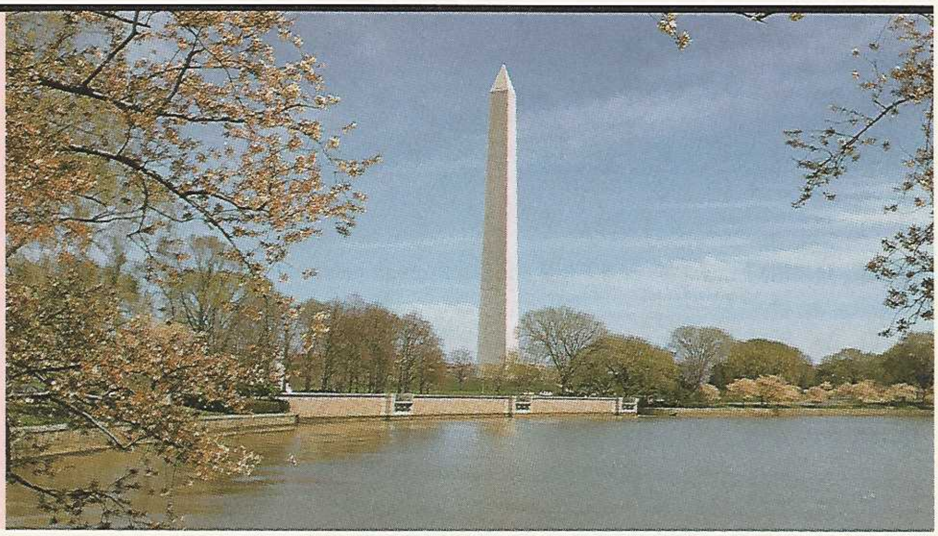
CHAPTER

10 Solutions of Quadratic Equations

A rock is dropped from the top of the Washington Monument. If the monument is 555 feet tall, how long will it take the rock to strike the ground?



10–1 ■ Solutions of quadratic equations by extracting the roots

In section 4–7, we solved quadratic equations of the form

$$ax^2 + bx + c = 0, a \neq 0$$

by factoring. It was necessary that the quadratic expression $ax^2 + bx + c$ be factorable to use the method discussed. Let us review the procedures used in solving quadratic equations by factoring.

To solve a quadratic equation by factoring

1. Write the equation in standard form

$$ax^2 + bx + c = 0, a \neq 0$$

if the equation is not written in this form.

2. Factor the expression $ax^2 + bx + c$.
3. Set each of the resulting factors involving the variable equal to 0 and solve each linear equation for the variable.
4. Check your solutions in the original equation.

■ Example 10–1 A

Find the solution set of each quadratic equation by factoring.

1. $x^2 - x - 12 = 0$

$$
\begin{aligned}
(x - 4)(x + 3) &= 0 && \text{Factor the left member}\\
x - 4 = 0 \quad \text{or} \quad x + 3 &= 0 && \text{Set each factor equal to 0}\\
x = 4 \qquad\qquad x &= -3 && \text{Solve each equation for } x
\end{aligned}
$$

The solution set is $\{-3,4\}$.

2. $3y^2 = 7y + 6$

$$
\begin{aligned}
3y^2 - 7y - 6 &= 0 && \text{Write the equation in standard form}\\
(3y + 2)(y - 3) &= 0 && \text{Factor the left member}\\
3y + 2 = 0 \quad \text{or} \quad y - 3 &= 0 && \text{Set each factor equal to 0}\\
3y = -2 \qquad\qquad y &= 3 && \text{Solve each equation for } y\\
y = -\frac{2}{3} \qquad\qquad y &= 3
\end{aligned}
$$

The solution set is $\left\{-\frac{2}{3},3\right\}$. ■

Extracting the roots

Given the quadratic equation $x^2 - 9 = 0$, factoring the left member and solving the resulting equations, we get

$$
\begin{aligned}
(x - 3)(x + 3) &= 0\\
x - 3 = 0 \quad \text{or} \quad x + 3 &= 0\\
x = 3 \quad \text{or} \quad x &= -3
\end{aligned}
$$

The solutions of the equation are 3 or -3.

We can obtain the same result if we write the equation in the form

$$x^2 = 9$$

Since 9 is positive, we can take the square root of each member of the equation. Then

$$x = \sqrt{9} = 3 \quad \text{or} \quad x = -\sqrt{9} = -3$$

and we obtain the same result. This development justifies the following method of solving a quadratic equation by **extracting the roots** using the **square root property**.

Square root property

If k is a nonnegative number and $x^2 = k$, then

$$x = \sqrt{k} \quad \text{or} \quad x = -\sqrt{k}$$

■ Example 10–1 B

Find the solution set of the following quadratic equations by extracting the roots.

1. $x^2 = 25$

$$
\begin{aligned}
x = \sqrt{25} \quad \text{or} \quad x &= -\sqrt{25} && \text{Extract the roots}\\
x = 5 \qquad\qquad x &= -5 && \sqrt{25} = 5
\end{aligned}
$$

The solution set is $\{-5,5\}$.

2. $y^2 = 18$

$$y = \sqrt{18} \quad \text{or} \quad y = -\sqrt{18}$$ Extract the roots
$$y = 3\sqrt{2} \qquad y = -3\sqrt{2}$$ $\sqrt{18} = 3\sqrt{2}$

The solution set is $\{-3\sqrt{2}, 3\sqrt{2}\}$.

3. $x^2 - 12 = 0$

$$x^2 = 12$$ Add 12 to each member
$$x = \sqrt{12} \quad \text{or} \quad x = -\sqrt{12}$$ Extract the roots
$$x = 2\sqrt{3} \qquad x = -2\sqrt{3}$$ $\sqrt{12} = 2\sqrt{3}$

The solution set is $\{-2\sqrt{3}, 2\sqrt{3}\}$.

4. $z^2 = -9$

Since -9 is negative and the property requires that k is a nonnegative number, we are not able to solve this equation in the set of real numbers. The equation has no solution so the solution set is $\emptyset$.

5. $2x^2 = 98$

To extract the roots, the squared term must have coefficient 1.

$$2x^2 = 98$$
$$x^2 = 49$$ Divide each term by 2
$$x = \sqrt{49} \quad \text{or} \quad x = -\sqrt{49}$$ Extract the roots
$$x = 7 \qquad x = -7$$ $\sqrt{49} = 7$

The solution set is $\{-7, 7\}$.

▶ ***Quick check*** Find the solution set of the equation $3x^2 = 24$ by extracting the roots. ■

Any equation that is written in the form

$$(x + q)^2 = k \text{ or } (px + q)^2 = k$$

can be solved by extracting the roots. Consider the following examples.

■ ***Example 10–1 C***

Find the solution set of the following quadratic equations by extracting the roots.

1. $(x - 2)^2 = 4$

$$x - 2 = \sqrt{4} \quad \text{or} \quad x - 2 = -\sqrt{4}$$ Extract the roots
$$x - 2 = 2 \qquad x - 2 = -2$$ $\sqrt{4} = 2$ or -2
$$x = 2 + 2 = 4 \qquad x = 2 - 2 = 0$$ Add 2 to each member

The solution set is $\{0, 4\}$.

2. $(2y - 1)^2 = 24$

$$2y - 1 = \sqrt{24} \quad \text{or} \quad 2y - 1 = -\sqrt{24}$$ Extract the roots
$$2y - 1 = 2\sqrt{6} \qquad 2y - 1 = -2\sqrt{6}$$ $\sqrt{24} = 2\sqrt{6}$
$$2y = 1 + 2\sqrt{6} \qquad 2y = 1 - 2\sqrt{6}$$ Add 1 to each member
$$y = \frac{1 + 2\sqrt{6}}{2} \qquad y = \frac{1 - 2\sqrt{6}}{2}$$ Divide each member by 2

The solution set is $\left\{\frac{1 - 2\sqrt{6}}{2}, \frac{1 + 2\sqrt{6}}{2}\right\}$.

▶ ***Quick check*** Find the solution set of the equation $(x + 4)^2 = 9$ by extracting roots.

Mastery points

Can you

- Solve a quadratic equation by factoring?
- Solve quadratic equations of the form $x^2 = k$ and $(px + q)^2 = k$ by extracting the roots?

Exercise 10–1

Find the solution set of each quadratic equation by extracting the roots or by factoring. Express radicals in simplest form. All variables represent nonnegative numbers. See examples 10–1 A and B.

Example $3x^2 = 24$

Solution

$x^2 = 8$ — Divide each member by 3

$x = \sqrt{8}$ or $x = -\sqrt{8}$ — Extract the roots

$x = 2\sqrt{2}$ $\quad x = -2\sqrt{2}$ — $\sqrt{8} = \sqrt{4 \cdot 2} = 2\sqrt{2}$

The solution set is $\{2\sqrt{2}, -2\sqrt{2}\}$.

1. $x^2 + 2x - 15 = 0$
2. $x^2 - 4x + 3 = 0$
3. $2y^2 - y - 6 = 0$
4. $5z^2 - 16z + 3 = 0$
5. $x^2 = 4$
6. $x^2 = 49$
7. $x^2 = 64$
8. $x^2 = 81$
9. $y^2 = 11$
10. $x^2 = 5$
11. $a^2 = 20$
12. $x^2 = 28$
13. $x^2 - 3 = 0$
14. $x^2 - 13 = 0$
15. $p^2 - 32 = 0$
16. $x^2 - 40 = 0$
17. $4x^2 = 36$
18. $5x^2 = 75$
19. $3z^2 = 18$
20. $5x^2 = 15$
21. $2x^2 - 100 = 0$
22. $4x^2 - 64 = 0$
23. $7x^2 - 56 = 0$
24. $9a^2 - 162 = 0$
25. $\frac{3}{4}x^2 - 6 = 0$
26. $\frac{1}{5}x^2 - \frac{3}{5} = 0$
27. $\frac{1}{3}x^2 = \frac{2}{3}$
28. $\frac{2}{3}x^2 = 8$
29. $\frac{5}{2}x^2 - \frac{3}{5} = 0$
30. $\frac{4x^2}{3} - 3 = 0$
31. $\frac{1}{2}y^2 - \frac{3}{2} = 4$
32. $\frac{z^2}{4} - 6 = \frac{3}{4}$

See example 10–1 C.

Example $(x + 4)^2 = 9$

Solution

$x + 4 = 3$ or $x + 4 = -3$ — Extract the roots

$x = -4 + 3$ $\quad x = -4 - 3$ — Add -4 to each member

$x = -1$ $\quad x = -7$

The solution set is $\{-1, -7\}$.

33. $(x + 2)^2 = 4$
34. $(x + 6)^2 = 16$
35. $(x - 4)^2 = 25$
36. $(x - 3)^2 = 49$
37. $(x + 3)^2 = 6$
38. $(x - 1)^2 = 7$
39. $(x - 9)^2 = 18$
40. $(x + 8)^2 = 8$
41. $(x + 5)^2 = 32$
42. $(x - 10)^2 = 27$
43. $(x + a)^2 = 36$
44. $(x - a)^2 = 50$
45. $(x - 6)^2 = a^2$
46. $(x + 7)^2 = m^2$
47. $(x - p)^2 = q^2$
48. $(x + 5)^2 = t^2$
49. $(2x - 3)^2 = 16$
50. $(3x + 2)^2 = 24$

Solve by setting up a quadratic equation and extracting the roots.

Example A square has an area of 16 square inches. Find the length of each side.

Solution Use the formula $A = s^2$, where A is the area and s is the length of a side. Then $16 = s^2$ or $s^2 = 16$, and

$s = \sqrt{16}$ or $s = -\sqrt{16}$ Exract the roots

$s = 4$ $\quad s = -4$ $\quad \sqrt{16} = 4$

Since a square cannot have a side that is -4 inches long, then $s = 4$ inches. The length of each side of the square is 4 inches.

51. Find the length of each side of a square whose area is 25 square meters.

52. Given a square whose area is 45 square centimeters, how long is each side of the square?

53. A circle has an area of approximately 12.56 square feet. Find the approximate length of the radius r of the circle if $A \approx 3.14r^2$.

54. Find the approximate length of the radius of a circle whose area is approximately 50.24 square yards. (Refer to exercise 53 for the formula.)

55. The square of a number less 81 is equal to zero. Find the number.

56. Four times the square of a number is 100. Find the number.

57. The square of a number is equal to nine times the number. Find the number.

58. If you subtract eight times a number from two times the square of the number, you get zero. Find the number.

59. The sum of the areas of two squares is 245 square inches. If the length of the side of the larger square is twice the length of the side of the smaller square, find the lengths of the sides of the two squares.

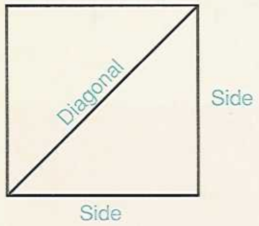
60. The length of the side of one square is three times the length of the side of a second square. If the difference in their areas is 128 square centimeters, find the lengths of the sides of the two squares.

61. The width of a rectangle is one-fourth of the length. If the area is 144 square meters, find the length of the rectangle. (*Hint:* $A = \ell w$.)

62. The length of a rectangle is three times the width. If the area of the rectangle is 147 square feet, find the dimensions of the rectangle.

63. The sum of the areas of two circles is 80π. Find the length of the radius of each circle if one radius is twice as long as the other.

Solve by using the relationship that exists for a square: The sum of the squares of two sides is equal to the square of the diagonal of the square.



$[(\text{side})^2 + (\text{side})^2 = (\text{diagonal})^2]$

64. Find the length of the side of a square whose diagonal is 16 inches long.

65. Find the length of the side of a square whose diagonal is 10 centimeters long.

66. Find the length of the side of a square whose diagonal is 24 feet long. Simplify the radical answer.

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Review exercises

Multiply the following. See section 3–2.

1. $(x - 2)^2$

2. $(3z + 2)^2$

Completely factor the following. See sections 4–2 and 4–3.

3. $x^2 + 18x + 81$

4. $9y^2 + 30y + 25$

Perform the indicated operations. See sections 6–1 and 6–2.

5. $\dfrac{3x}{x + 2} - \dfrac{x}{x^2 - 4}$

6. $\dfrac{x - 3}{x^2 - x - 2} \div \dfrac{x^2 - 9}{x + 1}$

10–2 ■ Solutions of quadratic equations by completing the square

Building perfect square trinomials

The methods we have used to solve quadratic equations thus far have applied to special cases of the quadratic equation. The method that we call **completing the square** involves transforming the quadratic equation

$$ax^2 + bx + c = 0$$

into the form

$$(x + q)^2 = k, k \geq 0$$

where q and k are constants. This latter equation can then be solved by extracting the roots, as we did in section 10–1.

Consider the following perfect square trinomials and their equivalent binomial squares.

$$\begin{aligned} x^2 + 2x + 1 &= (x + 1)(x + 1) = (x + 1)^2 \\ x^2 - 10x + 25 &= (x - 5)(x - 5) = (x - 5)^2 \\ x^2 - 14x + 49 &= (x - 7)(x - 7) = (x - 7)^2 \end{aligned}$$

In each of the perfect square trinomials in the left member,

a. the coefficient of x^2 is 1.
b. the third term, the constant, is the square of one-half of the coefficient of the variable x in the middle term.

We further observe that in the square of the binomial in the right member, the constant term in the binomial is one-half of the coefficient of the variable x in the middle term. That is,

1. In the trinomial $x^2 + 2x + 1$, the constant term, 1, is the square of one-half the coefficient of the middle term, 2. Thus,

$$\left[\frac{1}{2}(2)\right]^2 = (\underset{\uparrow}{1})^2 = 1$$

Constant term of the binomial

2. In the trinomial $x^2 - 10x + 25$, the constant term, 25, is the square of one-half of -10. Thus,

$$\left[\frac{1}{2}(-10)\right]^2 = (\underset{\uparrow}{-5})^2 = 25$$

Constant term of the binomial

3. In the trinomial $x^2 - 14x + 49$, the constant term, 49, is the square of one-half of -14. Thus,

$$\left[\frac{1}{2}(-14)\right]^2 = (\underset{\uparrow}{-7})^2 = 49$$

Constant term of the binomial

Now we can use these observations to "build" perfect square trinomials by **completing the square** and obtain the equivalent square of a binomial.

Example 10–2 A

Complete the square in each of the following expressions. Write the resulting expression as the square of a binomial.

1. $x^2 + 6x$

Since the coefficient of x is 6, the constant term is the square of one-half of 6.

$$\left[\frac{1}{2}(6)\right]^2 = (3)^2 = 9$$

The trinomial becomes

$$x^2 + 6x + 9$$

which factors into

$$(x + 3)(x + 3) = (x + 3)^2$$

2. $x^2 - 8x$

Since the coefficient of x is -8, the constant term is the square of one-half of -8.

$$\left[\frac{1}{2}(-8)\right]^2 = (-4)^2 = 16$$

The trinomial becomes

$$x^2 - 8x + 16$$

which factors into

$$(x - 4)(x - 4) = (x - 4)^2$$

3. $y^2 - 3y$

Since the coefficient of y is -3, the constant term is the square of one-half of -3.

$$\left[\frac{1}{2}(-3)\right]^2 = \left(-\frac{3}{2}\right)^2 = \frac{9}{4}$$

The trinomial becomes

$$y^2 - 3y + \frac{9}{4}$$

which factors into

$$\left(y - \frac{3}{2}\right)\left(y - \frac{3}{2}\right) = \left(y - \frac{3}{2}\right)^2$$

▶ ***Quick check*** Complete the square in the expression $x^2 + \frac{1}{3}x$. Write as the square of a binomial. ■

Solutions by completing the square

The following examples show how we use the previous procedure to find the solution set of a quadratic equation by **completing the square.**

■ ***Example 10–2 B***

Find the solution set by completing the square.

1. $x^2 - 2x - 8 = 0$

We first isolate the terms containing the variable in the left member.

$x^2 - 2x = 8$ — Add 8 to each member

Complete the square in the left member.

$\left[\frac{1}{2}(-2)\right]^2 = (-1)^2 = 1$ — Square one-half of the coefficient of x

$x^2 - 2x + 1 = 8 + 1$ — Add 1 to each member

$(x - 1)^2 = 9$ — Factor the left member and combine like terms in the right member

$x - 1 = \sqrt{9}$ or $x - 1 = -\sqrt{9}$ — Extract the roots

$x - 1 = 3$ $\quad$ $x - 1 = -3$ — $\sqrt{9} = 3$

$x = 1 + 3$ $\quad$ $x = 1 - 3$ — Add 1 to each member

$x = 4$ $\quad$ $x = -2$ — Combine in right member

The solution set is $\{-2,4\}$.

Check:

1. When $x = 4$

$x^2 - 2x - 8 = 0$

$(4)^2 - 2(4) - 8 = 0$ — Replace x with 4

$16 - 8 - 8 = 0$ — Order of operations

$0 = 0$ (True)

2. When $x = -2$

$x^2 - 2x - 8 = 0$

$(-2)^2 - 2(-2) - 8 = 0$ — Replace x with -2

$4 + 4 - 8 = 0$ — Order of operations

$0 = 0$ (True)

In future examples, we will not show a check but you should always do this.

2. $x^2 - 6x + 2 = 0$

Isolate the terms containing the variable in the left member.

$x^2 - 6x = -2$ — Add -2 to each member

Complete the square in the left member.

$$\left[\frac{1}{2}(-6)\right]^2 = (-3)^2 = 9$$ Square one-half of the coefficient of x

$$x^2 - 6x + 9 = -2 + 9$$ Add 9 to each member

$$(x - 3)^2 = 7$$ Factor the left member and combine like terms in the right member

$$x - 3 = \sqrt{7} \quad \text{or} \quad x - 3 = -\sqrt{7}$$ Extract the roots

$$x = 3 + \sqrt{7} \qquad x = 3 - \sqrt{7}$$ Add 3 to each member

The solution set is $\{3 - \sqrt{7}, 3 + \sqrt{7}\}$.

3. $4x^2 + 4x = 3$

To complete the square using the method we have described, it is necessary for the coefficient of x^2 to be 1. To get this, we divide each term of the equation by 4.

$$\frac{4x^2}{4} + \frac{4x}{4} = \frac{3}{4}$$ Divide each term by 4

$$x^2 + x = \frac{3}{4}$$ Reduce where possible

Complete the square in the left member.

$$\left[\frac{1}{2}(1)\right]^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$ Square one-half of the coefficient of x

$$x^2 + x + \frac{1}{4} = \frac{3}{4} + \frac{1}{4}$$ Add $\frac{1}{4}$ to each member

$$\left(x + \frac{1}{2}\right)^2 = 1$$ Factor the left member and combine like terms in the right member

$$x + \frac{1}{2} = \sqrt{1} \quad \text{or} \quad x + \frac{1}{2} = -\sqrt{1}$$ Extract the roots

$$x + \frac{1}{2} = 1 \qquad x + \frac{1}{2} = -1$$ $\sqrt{1} = 1$

$$x = -\frac{1}{2} + 1 \qquad x = -\frac{1}{2} - 1$$ Add $-\frac{1}{2}$ to each member

$$x = -\frac{1}{2} + \frac{2}{2} = \frac{1}{2} \qquad x = -\frac{1}{2} - \frac{2}{2} = -\frac{3}{2}$$ Combine like terms

The solution set is $\left\{\frac{1}{2}, -\frac{3}{2}\right\}$.

▶ ***Quick check*** Find the solution set of $x^2 + 7x - 2 = 0$ by completing the square.

To find the solution set of the quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$, by completing the square, we proceed as follows:

1. If $a = 1$, proceed to step 2. If $a \neq 1$, divide each term of the equation by a and simplify.
2. Write the equation with the variable terms in the left member and the constant term in the right member.
3. Add to each member of the equation the square of one-half of the numerical coefficient of the middle term of the original equation.
4. Write the left member as a trinomial and factor it as the square of a binomial. Combine like terms in the right member.
5. Extract the roots and solve the resulting linear equations.
6. Check the solutions in the original equation.

Mastery points

Can you

- Complete the square of an expression in the form $x^2 + bx$?
- Find the solution set of a quadratic equation by completing the square?

Exercise 10–2

Complete the square of each of the following and factor as the square of a binomial. See example 10–2 A.

Example $x^2 + \frac{1}{3}x$

Solution $\left[\frac{1}{2}\left(\frac{1}{3}\right)\right]^2 = \left(\frac{1}{6}\right)^2 = \frac{1}{36}$ Square one-half of the coefficient of x

$$x^2 + \frac{1}{3}x + \frac{1}{36} = \left(x + \frac{1}{6}\right)^2$$

1. $x^2 + 10x$ **2.** $x^2 + 4x$ **3.** $a^2 - 12a$ **4.** $y^2 - 18y$ **5.** $x^2 + 24x$

6. $b^2 + 16b$ **7.** $y^2 - 20y$ **8.** $x^2 - 22x$ **9.** $x^2 + x$ **10.** $x^2 + 5x$

11. $x^2 - 7x$ **12.** $y^2 - 9y$ **13.** $x^2 + \frac{1}{2}x$ **14.** $b^2 + \frac{1}{4}b$ **15.** $s^2 - \frac{1}{5}s$

16. $x^2 - \frac{3}{8}x$ **17.** $y^2 + \frac{2}{3}y$ **18.** $x^2 + \frac{3}{4}x$ **19.** $m^2 - \frac{2}{5}m$ **20.** $x^2 - \frac{1}{8}x$

21. $a^2 - \frac{3}{2}a$ **22.** $b^2 + \frac{5}{2}b$

Find the solution set by completing the square. See example 10–2 B.

Example $x^2 + 7x - 2 = 0$

Solution $x^2 + 7x = 2$ — Add 2 to each member

Complete the square in the left member.

$$\left[\frac{1}{2}(7)\right]^2 = \left(\frac{7}{2}\right)^2 = \frac{49}{4}$$ Square one-half of the coefficient of x

$$x^2 + 7x + \frac{49}{4} = 2 + \frac{49}{4}$$ Add $\frac{49}{4}$ to each member

$$\left(x + \frac{7}{2}\right)^2 = \frac{57}{4}$$ Factor the left member and combine in the right member

$$x + \frac{7}{2} = \sqrt{\frac{57}{4}} \quad \text{or} \quad x + \frac{7}{2} = -\sqrt{\frac{57}{4}}$$ Extract the roots

$$x + \frac{7}{2} = \frac{\sqrt{57}}{2} \qquad x + \frac{7}{2} = -\frac{\sqrt{57}}{2}$$ $\sqrt{\frac{57}{4}} = \frac{\sqrt{57}}{2}$

$$x = -\frac{7}{2} + \frac{\sqrt{57}}{2} \qquad x = -\frac{7}{2} - \frac{\sqrt{57}}{2}$$ Add $-\frac{7}{2}$ to each member

$$x = \frac{-7 + \sqrt{57}}{2} \qquad x = \frac{-7 - \sqrt{57}}{2}$$ Combine like terms

The solution set is $\left\{\frac{-7 - \sqrt{57}}{2}, \frac{-7 + \sqrt{57}}{2}\right\}$.

23. $x^2 + 8x + 7 = 0$
24. $x^2 + 12x + 11 = 0$
25. $a^2 - 4a - 12 = 0$
26. $y^2 - 10y + 9 = 0$
27. $x^2 - 4x = -3$
28. $x^2 + 14x = -13$
29. $u^2 - u - 1 = 0$
30. $a^2 + 3a - 1 = 0$
31. $x^2 - 5x + 2 = 0$
32. $n^2 - 3n - 2 = 0$
33. $y^2 - 4y = 81$
34. $b^2 - 12b = -25$
35. $h^2 + 21h + 10 = 0$
36. $3x^2 + 6x = 3$
37. $2x^2 + x - 3 = 0$
38. $3x^2 - 10x = -3$
39. $2y^2 + 7y + 3 = 0$
40. $3a^2 + 8a - 4 = 0$
41. $4x^2 - 4x = 3$
42. $2b^2 - b - 15 = 0$
43. $6a^2 - 13a = -6$
44. $6n^2 + n = 1$
45. $y^2 = 3 - y$
46. $x^2 + 1 = -3x$
47. $2 - a = 6a^2$
48. $4 - x^2 = 2x$
49. $-n^2 - 4 = -6n$
50. $1 - n^2 = 3n$
51. $2x^2 - 3 = x + 4$
52. $3a^2 + 5 = 4a + 8$
53. $(x + 3)(x - 2) = 1$
54. $(x - 5)(x - 3) = 4$
55. $3x(2x + 5) = -3$
56. $4x(x - 2) = 1$

Solve the following problems by setting up a quadratic equation and completing the square.

Example A piece of lumber is divided into two pieces so that one piece is 5 inches longer than the other. If the product of their lengths is 104 square inches, what is the length of each piece?

Solution Let x = the length of the shorter piece. Then $x + 5$ = the length of the longer piece.

the product of the lengths	is	104 square inches
$x(x + 5)$	$=$	104

The equation is $x(x + 5) = 104$.

$$x^2 + 5x = 104 \qquad \text{Multiply in the left member}$$

$$\left[\frac{1}{2}(5)\right]^2 = \left(\frac{5}{2}\right)^2 = \frac{25}{4} \qquad \text{Square one-half of the coefficient of } x$$

$$x^2 + 5x + \frac{25}{4} = 104 + \frac{25}{4} \qquad \text{Add } \frac{25}{4} \text{ to each member}$$

$$\left(x + \frac{5}{2}\right)^2 = \frac{416}{4} + \frac{25}{4} \qquad \text{Factor left member}$$

$$\left(x + \frac{5}{2}\right)^2 = \frac{441}{4} \qquad \text{Combine in the right member}$$

$$x + \frac{5}{2} = \sqrt{\frac{441}{4}} \quad \text{or} \quad x + \frac{5}{2} = -\sqrt{\frac{441}{4}} \qquad \text{Extract the roots}$$

$$x = -\frac{5}{2} + \frac{21}{2} \qquad x = -\frac{5}{2} - \frac{21}{2} \qquad \sqrt{441} = 21$$

$$x = \frac{-5 + 21}{2} = \frac{16}{2} = 8 \qquad x = \frac{-5 - 21}{2} = \frac{-26}{2} = -13 \qquad \text{Combine like terms}$$

The solution set of the equation is $\{8, -13\}$. Since we want the length of lumber, -13 is not an appropriate answer. Therefore, $x = 8$ and $x + 5 = 13$. The two pieces have lengths 8 inches and 13 inches.

57. A metal bar is to be divided into two pieces so that one piece is 4 inches shorter than the other. If the sum of the squares of the two lengths is 208 square inches, find the two lengths.

58. To find the total surface area of an automobile cylinder, we use the formula $A = 2\pi r^2 + 2\pi rh$, where π is approximately equal to the constant $\frac{22}{7}$. If the area A of the cylinder is approximately 88 square inches and the height h is 7 inches, find the approximate value of radius r.

59. One surface of a rectangular solid has a width w that is 8 millimeters shorter than its length ℓ. If the area A of the surface is 105 square millimeters, what are its dimensions? (*Hint:* $A = \ell w$.)

60. The length of a rectangular-shaped piece of paper is 7 inches longer than its width. What are the dimensions of the paper if it has an area of 78 square inches?

61. The perimeter of a rectangle is 52 inches and its area is 153 square inches. What are its dimensions? The formula for the perimeter of a rectangle is $P = 2\ell + 2w$. If we substitute 52 in place of P, we have $52 = 2\ell + 2w$. Divide each member of the equation by 2. Then $26 = \ell + w$. We can use this fact to establish the unknowns.
Width of rectangle: w
Length of rectangle: $26 - w$
Equation: $153 = w(26 - w)$

62. The perimeter of a rectangle is 38 centimeters and its area A is 88 square centimeters. What are its dimensions? (See exercise 61.)

63. The perimeter of a rectangle is 18 meters and its area is $19\frac{1}{4}$ square meters. What are its dimensions?

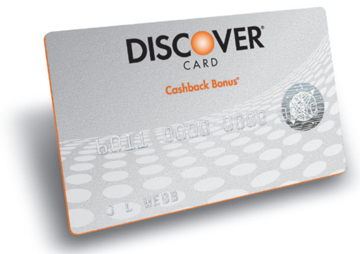
64. The area of a rectangular piece of sheet metal is 117 square inches. If the sum of the length ℓ and the width w is 22 inches, what are the dimensions of the metal plate?

65. A rectangular lot has an area of 84 square rods. If the sum of the length ℓ and the width w is 20 rods, what are the dimensions?

66. If two metal bars are the same length and if the length of one is increased by 3 centimeters and the second is decreased by 3 centimeters, the product of these two lengths is 27 square centimeters. Find the original lengths.

67. Two rectangular metal surfaces have the same width. If the width of one is increased by 6 inches and the other is increased by 8 inches, the product of the two widths is 99 square inches. Find the original widths.

68. If P dollars is invested at r percent compounded annually, at the end of two years it will grow to an amount $A = P(1 + r)^2$. At what rate will \$200 grow to \$224.72 in two years?



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Review exercises

Evaluate the following expressions for the given values. See sections 1–8 and 9–1.

1. $\sqrt{a+b}$; $a = 2$ and $b = 7$
2. $\sqrt{b^2 - 4ac}$; $a = -1$, $b = 5$, and $c = 5$
3. Find the solution set of the system of equations
   $2x - y = 4$
   $3x + 2y = 6$
   by any method.
   See section 8–3.
4. Graph the equation $4x - 3y = -12$.
   See section 7–1.
5. If 2 dozen oranges cost \$2.48, how many dozens of oranges can you buy for \$11.16? Set up a proportion. See section 5–4.

10–3 ■ Solutions of quadratic equations by the quadratic formula

Identifying a, b, and c in a quadratic equation

We have found solution sets of quadratic equations by factoring, extracting the roots, and by completing the square. Even though the solution set of *any* such quadratic equation can be found by completing the square, a general formula, which is called the **quadratic formula,** can be derived that will enable us to find the solution set in an easier fashion.

To use the quadratic formula, the equation must be written in standard form,

$$ax^2 + bx + c = 0, a > 0$$

and we must be able to identify the coefficients a, b, and c. In identifying a, b, and c, we note that

1. a is the coefficient of x^2
2. b is the coefficient of x
3. c is the constant term

■ Example 10–3 A

Write each quadratic equation in standard form and identify the values of a, b, and c.

1. $3x^2 - 2x + 1 = 0$
   The equation is in standard form.

   $$3x^2 - 2x + 1 = 0$$

   Constant term, $c = 1$
   Coefficient of x, $b = -2$
   Coefficient of x^2, $a = 3$

2. $3x^2 - 4 = x$
   The equation must be written in standard form.

   $3x^2 - x - 4 = 0$ Add $-x$ to each member

   $a = 3$, $b = -1$, $c = -4$

3. $4x(x - 3) = 2x - 1$

The equation must be written in standard form.

$$4x^2 - 12x = 2x - 1 \qquad \text{Multiply in left member}$$
$$4x^2 - 14x + 1 = 0 \qquad \text{Add } -2x \text{ and 1 to each member}$$
$$a = 4,\ b = -14,\ c = 1$$

▶ ***Quick check*** Write the quadratic equation $2x^2 = 4 - 5x$ in standard quadratic form and identify the values of a, b, and c.

Solving quadratic equations using the quadratic formula

To derive the quadratic formula, we solve the equation $ax^2 + bx + c = 0$ by completing the square.

$$ax^2 + bx + c = 0,\ a > 0$$

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0 \qquad \text{Divide each term of the equation by } a$$

$$x^2 + \frac{b}{a}x = -\frac{c}{a} \qquad \text{Subtract } \frac{c}{a} \text{ from each member}$$

$$\left[\frac{1}{2}\left(\frac{b}{a}\right)\right]^2 = \left(\frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} \qquad \text{Square one-half of the coefficient of } x$$

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = -\frac{c}{a} + \frac{b^2}{4a^2} \qquad \text{Add } \frac{b^2}{4a^2} \text{ to each member}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a} \qquad \text{Write left member as the square of a binomial and change the order of terms in the right member}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{4ac}{4a^2} \qquad \text{Subtract fractions in the right member}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

$$x + \frac{b}{2a} = \sqrt{\frac{b^2 - 4ac}{4a^2}} \quad \text{or} \quad x + \frac{b}{2a} = -\sqrt{\frac{b^2 - 4ac}{4a^2}} \qquad \text{Extract the roots}$$

$$x + \frac{b}{2a} = \frac{\sqrt{b^2 - 4ac}}{\sqrt{4a^2}} \quad \text{or} \quad x + \frac{b}{2a} = -\frac{\sqrt{b^2 - 4ac}}{\sqrt{4a^2}}$$

$$x + \frac{b}{2a} = \frac{\sqrt{b^2 - 4ac}}{2a} \quad \text{or} \quad x + \frac{b}{2a} = -\frac{\sqrt{b^2 - 4ac}}{2a} \qquad \sqrt{4a^2} = 2a, \text{ since } a > 0$$

$$x = \frac{-b}{2a} + \frac{\sqrt{b^2 - 4ac}}{2a} \quad \text{or} \quad x = \frac{-b}{2a} - \frac{\sqrt{b^2 - 4ac}}{2a} \qquad \text{Subtract } \frac{b}{2a} \text{ from each member}$$

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \text{or} \quad x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

By combining the expressions, these results can be summarized by the **quadratic formula.**

Quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Note We read $\pm$ "plus or minus," which allows us to write the two solutions

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \text{or} \quad x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

as a single statement.

Note When writing the quadratic formula, be sure that the fraction bar extends all the way beneath the numerator.

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Fraction bar

A common mistake is to write this as

$$-b \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

To solve a quadratic equation by the quadratic formula

1. Write the equation in standard form, if it is not already in this form. ($a > 0$).
2. Identify a (coefficient of x^2), b (coefficient of x), and c (the constant).
3. Substitute the values of a, b, and c into the quadratic formula.
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
4. Simplify the resulting expressions.

■ Example 10–3 B

Find the solution set using the quadratic formula.

1. $x^2 - 2x - 8 = 0$

The equation is already in standard form where $a = 1$, $b = -2$, and $c = -8$.

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-8)}}{2(1)}$$ Replace a with 1, b with -2, and c with -8 in the quadratic formula

$$x = \frac{2 \pm \sqrt{4 - (-32)}}{2}$$ Simplify by performing indicated operations

$$x = \frac{2 \pm \sqrt{36}}{2}$$

$$x = \frac{2 \pm 6}{2}$$ $\sqrt{36} = 6$

$$x = \frac{2 + 6}{2} = \frac{8}{2} = 4 \quad \text{or} \quad x = \frac{2 - 6}{2} = \frac{-4}{2} = -2$$

The solution set is $\{-2,4\}$.

2. $x^2 = 4 - x$

Write the equation in standard form: $x^2 + x - 4 = 0$. Then $a = 1$, $b = 1$, and $c = -4$.

Substitute these values into the quadratic formula.

$$x = \frac{-1 \pm \sqrt{(1)^2 - 4(1)(-4)}}{2(1)}$$ Replace a with 1, b with 1, and c with -4

$$x = \frac{-1 \pm \sqrt{1 + 16}}{2}$$

$$x = \frac{-1 \pm \sqrt{17}}{2}$$

Then $x = \dfrac{-1 + \sqrt{17}}{2}$ or $x = \dfrac{-1 - \sqrt{17}}{2}$.

The solution set is $\left\{\dfrac{-1 + \sqrt{17}}{2}, \dfrac{-1 - \sqrt{17}}{2}\right\}$.

3. $x^2 - 7 = 0$

The equation can be written $x^2 + 0x - 7 = 0$, so $a = 1$, $b = 0$, and $c = -7$.

Substitute these values.

$$x = \frac{-0 \pm \sqrt{(0)^2 - 4(1)(-7)}}{2(1)}$$ Replace a with 1, b with 0, and c with -7

$$x = \frac{\pm \sqrt{28}}{2}$$

$$x = \pm\frac{2\sqrt{7}}{2}$$

$$x = \pm\sqrt{7}$$ Simplify by reducing

Thus $x = \sqrt{7}$ or $x = -\sqrt{7}$.

The solution set is $\{\sqrt{7}, -\sqrt{7}\}$.

4. $4x^2 - 3x = 0$

The equation could be written $4x^2 - 3x + 0 = 0$, so $a = 4$, $b = -3$, and $c = 0$.

Substitute these values.

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(4)(0)}}{2(4)}$$ Replace a with 4, b with -3, and c with 0

$$x = \frac{3 \pm \sqrt{9 - 0}}{8}$$

$$x = \frac{3 \pm \sqrt{9}}{8}$$

$$x = \frac{3 \pm 3}{8}$$

$$x = \frac{3 + 3}{8} = \frac{3}{4} \quad \text{or} \quad x = \frac{3 - 3}{8} = \frac{0}{8} = 0$$

The solution set is $\left\{\dfrac{3}{4}, 0\right\}$.

▶ ***Quick check*** Find the solution set.

a. $5x^2 - 2 = 3x$ b. $3 - \dfrac{2}{3}x^2 - \dfrac{7}{3}x = 0$ ■

Problem solving

Many useful formulas in the physical world have a second-degree term and are solved using the methods for quadratic equations. The following example illustrates this and some more application problems that require a quadratic equation to solve.

■ Example 10–3 C

1. The position of a particle moving on a straight line at time t in seconds is given by

   $s = 3t^2 - 5t(t > 0)$

   where s is the distance from the starting point in feet. How many seconds will it take to move the particle 8 feet in a positive direction?

   We want t when $s = 8$ feet.

   $$(8) = 3t^2 - 5t \quad \text{Replace } s \text{ with } 8$$
   $$3t^2 - 5t - 8 = 0 \quad \text{Write equation in standard form}$$
   $$(3t - 8)(t + 1) = 0 \quad \text{Factor left member}$$
   $$3t - 8 = 0 \text{ or } t + 1 = 0 \quad \text{Set each factor equal to 0}$$
   $$t = \frac{8}{3} \qquad t = -1 \quad \text{Solve each equation}$$

   Since we do not want a negative answer, the particle will move 8 feet in $\frac{8}{3}\left(\text{or } 2\frac{2}{3}\right)$ seconds.

   Set up quadratic equation and solve the following problems.

2. Find two consecutive whole numbers whose product is 156.

   Let $n =$ the lesser whole number. Then $n + 1 =$ the next consecutive whole number.

   $$\underbrace{n \cdot (n + 1)}_{\text{product of consecutive whole numbers}} \underbrace{=}_{\text{is}} \underbrace{156}_{156}$$

   $$n(n + 1) = 156$$
   $$n^2 + n = 156 \quad \text{Multiply in the left member}$$
   $$n^2 + n - 156 = 0 \quad \text{Add } -156 \text{ to each member}$$

   We will use the quadratic formula.

   $$n = \frac{-1 \pm \sqrt{(1)^2 - 4(1)(-156)}}{2(1)} \quad \text{Replace } a \text{ with 1, } b \text{ with 1, and } c \text{ with } -156$$
   $$n = \frac{-1 \pm \sqrt{1 + 624}}{2}$$
   $$n = \frac{-1 \pm \sqrt{625}}{2}$$
   $$n = \frac{-1 \pm 25}{2} \quad \sqrt{625} = 25$$
   $$n = \frac{-1 + 25}{2} = 12 \text{ or } n = \frac{-1 - 25}{2} = -13$$

   We reject the negative number since n is a whole number. So $n = 12$ and $n + 1 = 13$. The consecutive whole numbers are 12 and 13.

3. The sum of a number and its reciprocal is $\frac{25}{12}$. Find the number and its reciprocal.

Let n = the number. Then $\frac{1}{n}$ = the reciprocal of the number.

the sum of a number and its reciprocal	is	$\frac{25}{12}$
$n + \frac{1}{n}$	$=$	$\frac{25}{12}$

$$n + \frac{1}{n} = \frac{25}{12}$$

$$12n \cdot n + 12n \cdot \frac{1}{n} = 12n \cdot \frac{25}{12}$$ Multiply each member by $12n$ (the LCD) to clear the fractions

$$12n^2 + 12 = 25n$$ Reduce in each term

$$12n^2 - 25n + 12 = 0$$ Write in standard form

$$(3n - 4)(4n - 3) = 0$$ Factor left member

$$3n - 4 = 0 \text{ or } 4n - 3 = 0$$ Set each factor equal to 0 and solve

$$n = \frac{4}{3} \qquad n = \frac{3}{4}$$ Solve each equation

The number is $\frac{3}{4}$ and its reciprocal is $\frac{4}{3}$ or the number is $\frac{4}{3}$ and its reciprocal is $\frac{3}{4}$.

Mastery points

Can you

- Identify the values of a, b, and c in any quadratic equation?
- Use the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

to solve any quadratic equation?

Exercise 10–3

Write each quadratic equation in standard form and identify the values of a, b, and c, $a > 0$. See example 10–3 A.

Example $2x^2 = 4 - 5x$

Solution Add $-4 + 5x$ to each member to get the equation in standard form

$2x^2 + 5x - 4 = 0$

Then $a = 2$, $b = 5$, and $c = -4$.

1. $5x^2 - 3x + 8 = 0$
2. $4x^2 + x - 2 = 0$
3. $-6z^2 - 2z + 1 = 0$
4. $-3x^2 + x + 9 = 0$
5. $4x^2 = 2x - 1$
6. $y^2 = 5y + 3$
7. $x^2 = -3x$
8. $4x - 3x^2 = 0$
9. $5x^2 = 2$

10. $-8b^2 = -3$
11. $p(p + 3) = 4$
12. $2x(x - 9) = 1$
13. $(x + 3)(x - 1) = 6$
14. $(z - 4)(2z + 1) = -6$
15. $8m^2 - (m + 3) = 2m - 1$
16. $3x^2 - (2x - 5) = x - 6$

Find the solution set, using the quadratic formula. See example 10–3 B.

Example $5x^2 - 2 = 3x$

Solution Write the equation in standard form by adding $-3x$ to each member.

$5x^2 - 3x - 2 = 0$

Then $a = 5$, $b = -3$, and $c = -2$.

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(5)(-2)}}{2(5)}$$ Replace a with 5, b with -3, and c with -2

$$x = \frac{3 \pm \sqrt{9 + 40}}{10}$$

$$x = \frac{3 \pm \sqrt{49}}{10}$$

$$x = \frac{3 \pm 7}{10}.$$

$$x = \frac{3 + 7}{10} = 1 \text{ or } x = \frac{3 - 7}{10} = -\frac{4}{10} = -\frac{2}{5}$$

The solution set is $\left\{1, -\frac{2}{5}\right\}$.

17. $x^2 - 3x + 2 = 0$
18. $y^2 + 6y + 9 = 0$
19. $a^2 - 2a + 1 = 0$
20. $x^2 + 10x + 24 = 0$
21. $x^2 - 25 = 0$
22. $2x^2 - 8 = 0$
23. $5x^2 - 10 = 0$
24. $3x^2 - 9 = 0$
25. $-x^2 = -3x$
26. $x^2 = 4x$
27. $5x^2 - 9x = 0$
28. $2x^2 = 7x$
29. $x^2 - 9x + 4 = 0$
30. $a^2 - 5a = 6$
31. $x^2 + 2x - 6 = 0$
32. $y^2 + y - 1 = 0$
33. $a^2 + 1 = 8a$
34. $2x^2 = 7x - 6$
35. $3y^2 = 5y + 6$
36. $4t^2 = 8t - 3$
37. $3a^2 = -9a - 2$
38. $x^2 - 9x = 6$
39. $3r^2 = r + 10$
40. $3a^2 + 5a = 4$
41. $x^2 + 8x + 16 = 0$
42. $2v^2 + 5v = -2$
43. $4x^2 + 25 = 20x$
44. $4x^2 - 7 = 12x$
45. $4x^2 + 12x + 9 = 0$
46. $4t^2 = 9t + 6$
47. $3a^2 - 2a - 7 = 0$
48. $4x^2 = 8 - 2x$
49. $3x^2 = 18 - 6x$
50. $9x^2 + 4 = 12x$
51. $3r^2 - 3r = 8$

Example $3 - \frac{2}{3}x^2 - \frac{7}{3}x = 0$

Solution $9 - 2x^2 - 7x = 0$ Multiply each member by the LCD, 3

Write the equation in standard form.

$2x^2 + 7x - 9 = 0$

Then $a = 2$, $b = 7$, $c = -9$.

$$x = \frac{-(7) \pm \sqrt{(7)^2 - 4(2)(-9)}}{2(2)}$$ Replace a with 2, b with 7, and c with -9

$$x = \frac{-7 \pm \sqrt{49 + 72}}{4}$$
$$x = \frac{-7 \pm \sqrt{121}}{4}$$
$$x = \frac{-7 \pm 11}{4}$$
$$x = \frac{-7 + 11}{4} = \frac{4}{4} = 1 \text{ or } x = \frac{-7 - 11}{4} = -\frac{18}{4} = -\frac{9}{2}$$

The solution set is $\left\{-\frac{9}{2}, 1\right\}$.

52. $a^2 + a = \frac{15}{4}$

53. $y^2 - y = \frac{3}{5}$

54. $2x^2 - \frac{7}{2} + \frac{x}{2} = 0$

55. $\frac{2}{3}x^2 - x = \frac{4}{3}$

56. $\frac{3}{4}x^2 - \frac{1}{2}x - 4 = 0$

57. $\frac{1}{3}x^2 - \frac{3}{2} = \frac{1}{2}x$

58. $\frac{2}{3}y^2 - \frac{4}{9}y = \frac{1}{3}$

59. $\frac{2a}{3} = \frac{2}{9} - a^2$

Solve the following problems using methods for solving quadratic equations. See example 10–3 C–1.

60. Use the formula $s = vt + \frac{1}{2}at^2$, where s is the distance traveled, v is the velocity, t is the time, and a is the acceleration of an object. Find t when (a) $s = 8$, $v = 3$, $a = 4$; (b) $s = 5$, $v = 4$, $a = 2$.

61. The distance s through which an object will fall in t seconds is $s = \frac{1}{2}gt^2$ feet, where $g = 32$ ft/sec^2. Find t (correct to tenth of a second) when (a) $s = 64$, (b) $s = 96$, (c) $s = 120$.

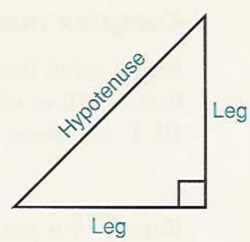
62. If a certain projectile is fired vertically into the air, the distance in feet above the ground in t seconds is given by $s = 160t - 16t^2$. Find t when (a) $s = 0$; (b) $s = 1{,}600$; (c) $s = 160$.

63. In a certain electric circuit, the relationship between i (in amperes), E (in volts), and R (in ohms) is given by $i^2R + iE = 8{,}000$. Find i ($i > 0$) when (a) $R = 2$ and $E = 80$, (b) $R = 4$ and $E = 60$.

64. A triangular-shaped plate has an altitude that is 5 inches longer than its base. If the area of the plate is 52 square inches, what is the length of the base b and the altitude h if the area of a triangle, A, is given by $A = \frac{1}{2}bh$?

65. The area of a triangle is 135 square inches. If the altitude is one-third the base, what are the lengths of the altitude and base? (Area $= \frac{1}{2}$ times base times altitude.)

66. A metal strip is shaped into a right triangle. In any right triangle, $c^2 = a^2 + b^2$, where c is the longest side, or hypotenuse, and a and b are the lengths of the other two sides, called legs. Find x when $a = x$, $b = x + 14$, and $c = x + 16$. (*Hint:* Substitute for a, b, and c in the above relationship and solve for x.)



67. The hypotenuse of a right triangle is 10 millimeters long. One leg is 2 millimeters longer than the other. What are the lengths of the two legs? (Refer to exercise 66 for information about the hypotenuse and legs of a right triangle.)

68. The lengths of the legs of a right triangle are consecutive integers. If the hypotenuse is 5 centimeters long, what are the lengths of the legs of the triangle?

69. One leg of a right triangle is 2 feet longer than the other leg. If the hypotenuse is 4 inches long, what are the lengths of the legs of the triangle?

70. A 15-foot ladder is leaning against a building. If the base of the ladder is 6 feet from the base of the building, how high up the building does the ladder reach?

71. Joe leans a 50-foot ladder against his house. If the top of the ladder is 45 feet above the ground, how far out is the foot of the ladder from the house?

See example 10–3 C–2 and 3.

72. Find two consecutive whole numbers whose product is 210.

73. Find two consecutive negative even integers whose product is 224. (*Hint:* Use n and $n + 2$.)

$\boxed{74.}$ Find two consecutive odd positive integers whose product is 143.

75. The sum of a number and its reciprocal is $\frac{50}{7}$. What is the number?

76. The sum of a number and its reciprocal is $\frac{61}{30}$. What is the number?

Review exercises

Perform the indicated operations. See sections 2–3 and 3–2.

1. $(4x^2 + 2x - 1) - (x^2 + x - 6)$
2. $(5y - 2)(y + 7)$
3. $(4z - 3)(4z + 3)$
4. $(3x - 5)^2$

Find the solution set of the following quadratic equations. See sections 10–1 and 10–3.

5. $3y^2 = 12$
6. $2x^2 - 7x - 4 = 0$
7. $x^2 - x = 10$

Subtract as indicated. See section 6–2.

8. $\dfrac{x + 2}{x^2 - 4} - \dfrac{x - 1}{x^2 - x - 6}$

10–4 ■ Complex solutions to quadratic equations

Complex numbers

Recall given the equation $x^2 = a$, we placed the restriction that $a \geq 0$. Suppose that $a < 0$ as in the equation $x^2 = -9$. Extracting the roots as we did in section 10–1, we obtain

$$x = \sqrt{-9} \quad \text{or} \quad x = -\sqrt{-9}$$

But $\sqrt{-9}$ is not a real number since there is no real number whose square is -9. Thus, in the set of real numbers, the equation $x^2 = -9$ does not have a solution. We introduce a new set of numbers called the *complex numbers*. To define this set, we need the definition of i.

Definition of i

The number i is defined by

$$i = \sqrt{-1}$$

We can now write

1. $\sqrt{-9} = \sqrt{9 \cdot -1} = \sqrt{9} \cdot \sqrt{-1} = 3i$
2. $\sqrt{-49} = \sqrt{49 \cdot -1} = \sqrt{49} \cdot \sqrt{-1} = 7i$

Observe two facts about the number i.

1. i is *not* a real number.
2. $i^2 = -1$ since $i = \sqrt{-1}$ and $i^2 = (\sqrt{-1})^2 = -1$.

We can now define a complex number.

Definition of a complex number

A complex number is any number that can be written in the form

$$a + bi \quad \text{or} \quad a - bi$$

where a and b are real numbers and $i = \sqrt{-1}$.

We call $a + bi$ and $a - bi$ the standard forms of a complex number.

■ Example 10–4 A

The following are complex numbers.

1. $2 + 3i$, where $a = 2$ and $b = 3$

2. $4 - 2i$, where $a = 4$ and $b = -2$

3. $5i$ since $5i = 0 + 5i$, where $a = 0$ and $b = 5$

4. $\sqrt{-4}$ since $\sqrt{-4} = \sqrt{4 \cdot -1} = \sqrt{4} \cdot \sqrt{-1} = 2i = 0 + 2i$ where $a = 0$ and $b = 2$

5. $2 + \sqrt{-5}$ since $2 + \sqrt{-5} = 2 + \sqrt{5 \cdot -1} = 2 + \sqrt{5} \cdot i = 2 + i\sqrt{5}$, where $a = 2$ and $b = \sqrt{5}$

6. 7 since $7 = 7 + 0i$ where $a = 7$ and $b = 0$

▶ ***Quick check*** Write $3 + \sqrt{-49}$ as a complex number $a + bi$. ■

From the last example, we can see that all real numbers are complex numbers, so the set of real numbers is a subset of the set of complex numbers.

Addition and subtraction of complex numbers

We add and subtract complex numbers in the same manner that we add and subtract polynomials. That is, we combine the similar terms.

■ Example 10–4 B

Combine the following complex numbers. Write the answer in standard form $a + bi$ or $a - bi$.

1. $(2 + 3i) + (4 - 5i)$

$= (2 + 4) + (3i - 5i)$ — Commutative and associative properties

$= 6 + (-2i)$ — Combine like terms.

$= 6 - 2i$

2. $(3 + i) - (4 - 3i)$

$= 3 + i - 4 + 3i$ — Definition of subtraction

$= (3 - 4) + (i + 3i)$ — Commutative and associative properties

$= -1 + 4i$ — Combine like terms

▶ ***Quick check*** Subtract $(4 - 2i) - (6 + 5i)$ ■

To multiply complex numbers, we apply the distributive property as we did when multiplying polynomials.

■ Example 10–4 C

Multiply the following complex numbers. Write the answer in standard form $a + bi$ or $a - bi$.

1. $2i(3 + 4i)$

$= 2i(3) + 2i(4i)$ — Distributive property

$= 6i + 8i^2$ — Multiply

$= 6i + 8(-1)$ — $i^2 = -1$

$= -8 + 6i$ — Write in form $a + bi$

2. $(1 - 4i)(2 + 5i)$

$= 1(2 + 5i) - 4i(2 + 5i)$ — Distributive property

$= 1(2) + 1(5i) + (-4i)(2) + (-4i)(5i)$

$= 2 + 5i - 8i - 20i^2$ — Multiply

$= 2 + 5i - 8i - 20(-1)$ — $i^2 = -1$

$= 2 + 5i - 8i + 20$

$= (2 + 20) + (5i - 8i)$ — Combine like terms

$= 22 + (-3i)$

$= 22 - 3i$

Note In this example, we should be reminded once again of the FOIL method for multiplying binomials. Remember, whenever i^2 appears, it must be replaced with -1.

3. $(5 + 2i)(5 - 2i)$

$= 5(5 - 2i) + 2i(5 - 2i)$ — Distributive property

$= 25 - 10i + 10i - 4i^2$

$= 25 - 4(-1)$ — Combine like terms

$= 25 + 4$

$= 29$

▶ ***Quick check*** Multiply $(2 - 3i)^2$ ■

Given polynomials $a + b$ and $a - b$, we call these *conjugates* of one another and know that $(a + b)(a - b) = a^2 - b^2$. In like fashion, $5 + 2i$ and $5 - 2i$ are called *complex conjugates* of one another. We found that

$$(5 + 2i)(5 - 2i) = 5^2 + 2^2 = 25 + 4 = 29$$

In general,

$$(a + bi)(a - bi) = a^2 + b^2$$

Thus, we find that the product of a complex number and its conjugate yields a *real number*. We use this property of complex numbers to rationalize the denominator of an indicated division of two complex numbers.

Example 10–4 D

Rationalize the denominator. Write the answer in standard form $a + bi$ or $a - bi$.

1. $\dfrac{4i}{2 + i}$

$= \dfrac{4i}{2 + i} \cdot \dfrac{2 - i}{2 - i}$ — Multiply the numerator and the denominator by the conjugate of $2 + i$, which is $2 - i$

$= \dfrac{4i(2 - i)}{(2 + i)(2 - i)}$

$= \dfrac{8i - 4i^2}{2^2 + 1^2}$ — Distributive property

$= \dfrac{8i - 4(-1)}{4 + 1}$ — $i^2 = -1$

$= \dfrac{4 + 8i}{5}$

$= \dfrac{4}{5} + \dfrac{8}{5}i$ — Divide each term of the numerator by 5

2. $\dfrac{4 + 3i}{3 - 5i}$

$= \dfrac{4 + 3i}{3 - 5i} \cdot \dfrac{3 + 5i}{3 + 5i}$ — Multiply the numerator and the denominator by the conjugate of $3 - 5i$, which is $3 + 5i$

$= \dfrac{(4 + 3i)(3 + 5i)}{(3 - 5i)(3 + 5i)}$

$= \dfrac{12 + 20i + 9i + 15i^2}{3^2 + 5^2}$

$= \dfrac{12 + 29i + 15(-1)}{9 + 25}$

$= \dfrac{-3 + 29i}{34}$

$= -\dfrac{3}{34} + \dfrac{29}{34}i$ — Divide each term of the numerator by 34

▶ ***Quick check*** Rationalize the denominator. $\dfrac{3 - 2i}{1 + i}$. Write the answer in standard form $a + bi$ or $a - bi$.

Quadratic equations with complex solutions

When solving equations of the form $ax^2 + bx + c = 0$, the solutions can be determined by the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

In our work thus far, we obtained rational or irrational solutions and the radicand, $b^2 - 4ac$, was always positive or zero. When $b^2 - 4ac < 0$ (negative), we obtain complex solutions of the quadratic equations.

Example 10–4 E Find the solution set of the following quadratic equations.

1. $(x - 3)^2 = -4$

This is in the form $(x + q)^2 = k$.

$$x - 3 = \pm\sqrt{-4} \quad \text{Extract the roots}$$
$$x - 3 = \pm 2i \quad \sqrt{-4} = 2i$$
$$x = 3 \pm 2i \quad \text{Add 3 to each member}$$

The solution set is $\{3 + 2i, 3 - 2i\}$.

Note We could have solved the problem by expanding the left member, writing the equation in standard form, and using the quadratic formula.

2. $x^2 - 3x = -7$

Write the equation in standard form and use the quadratic formula.

$$x^2 - 3x + 7 = 0 \quad \text{Add 7 to each member}$$

Now $a = 1$, $b = -3$, and $c = 7$.

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(7)}}{2(1)} \quad \text{Replace } a \text{ with 1, } b \text{ with } -3\text{, and } c \text{ with 7}$$
$$= \frac{3 \pm \sqrt{9 - 28}}{2} \quad \text{Perform indicated operations}$$
$$= \frac{3 \pm \sqrt{-19}}{2}$$
$$= \frac{3 \pm i\sqrt{19}}{2} \quad \sqrt{-19} = \sqrt{-1 \cdot 19} = \sqrt{-1} \cdot \sqrt{19} = i\sqrt{19}$$

The solution set is $\left\{\frac{3 + i\sqrt{19}}{2}, \frac{3 - i\sqrt{19}}{2}\right\}$.

3. $(x + 1)(2x - 3) = -8$

$$2x^2 - x - 3 = -8 \quad \text{Multiply in the left member}$$
$$2x^2 - x + 5 = 0 \quad \text{Write in standard form}$$

Then $a = 2$, $b = -1$, and $c = 5$. Use the quadratic formula

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(5)}}{2(2)} \quad \text{Replace } a \text{ with 2, } b \text{ with } -1\text{, and } c \text{ with 5}$$
$$= \frac{1 \pm \sqrt{1 - 40}}{4} \quad \text{Perform indicated operations}$$
$$= \frac{1 \pm \sqrt{-39}}{4}$$
$$= \frac{1 \pm i\sqrt{39}}{4} \quad \sqrt{-39} = i\sqrt{39}$$

The solution set is $\left\{\frac{1 + i\sqrt{39}}{4}, \frac{1 - i\sqrt{39}}{4}\right\}$.

▶ ***Quick check*** Find the solution set. $2y^2 - y = -5$

Because the expression $b^2 - 4ac$ determines the type of solutions a quadratic equation will have, we call $b^2 - 4ac$ the *discriminant*.

Thus when

1. $b^2 - 4ac > 0$, the equation has *two* distinct rational solutions if $b^2 - 4ac$ is a perfect square or two distinct irrational solutions if $b^2 - 4ac$ is *not* a perfect square.
2. $b^2 - 4ac = 0$, the equation has *one* rational solution.
3. $b^2 - 4ac < 0$, the equation has *two* complex solutions.

Example 10–4 F

Determine the type of solution(s) that the following quadratic equations yield by using the discriminant.

1. $x^2 - x - 6 = 0$

Since $a = 1$, $b = -1$, and $c = -6$, then

$$\begin{aligned} b^2 - 4ac &= (-1)^2 - 4(1)(-6) && \text{Replace } a \text{ with } 1,\ b \text{ with } -1, \text{ and } c \text{ with } -6 \\ &= 1 + 24 \\ &= 25 \end{aligned}$$

Then $b^2 - 4ac > 0$ and, since 25 is a perfect square, we would obtain *two distinct rational* solutions.

2. $3y^2 + 2y - 2 = 0$

Since $a = 3$, $b = 2$, and $c = -2$, then

$$\begin{aligned} b^2 - 4ac &= (2)^2 - 4(3)(-2) && \text{Replace } a \text{ with } 3,\ b \text{ with } 2, \text{ and } c \text{ with } -2 \\ &= 4 + 24 \\ &= 28 \end{aligned}$$

Then $b^2 - 4ac > 0$ and, since 28 is *not* a perfect square, we would obtain *two distinct irrational* solutions.

3. $x^2 - 10x + 25 = 0$

Since $a = 1$, $b = -10$, and $c = 25$, then

$$\begin{aligned} b^2 - 4ac &= (-10)^2 - 4(1)(25) \\ &= 100 - 100 \\ &= 0 \end{aligned}$$

Then $b^2 - 4ac = 0$ and we would obtain *one rational* solution.

4. $2y^2 + 2y + 7 = 0$

Since $a = 2$, $b = 2$, and $c = 7$, then

$$\begin{aligned} b^2 - 4ac &= (2)^2 - 4(2)(7) \\ &= 4 - 56 \\ &= -52 \end{aligned}$$

Then $b^2 - 4ac < 0$ and we would obtain *two complex* solutions.

► ***Quick check*** Determine the type of solutions $3x^2 + 2x = 4$ would yield by using the discriminant.

Mastery points

Can you

- Write a complex number in the standard form, $a + bi$?
- Add, subtract, and multiply complex numbers?
- Rationalize the denominator of an indicated quotient of two complex numbers?
- Find the complex solutions of a quadratic equation?
- Determine the type of solutions of any quadratic equation?

Exercise 10–4

Write the following complex numbers in standard form, $a + bi$ or $a - bi$. See example 10–4 A.

Example $3 + \sqrt{-49}$

Solution
$$\begin{aligned} &= 3 + \sqrt{49 \cdot -1} \\ &= 3 + \sqrt{49} \cdot \sqrt{-1} && \sqrt{ab} = \sqrt{a} \cdot \sqrt{b} \\ &= 3 + 7i && \sqrt{49} = 7;\ \sqrt{-1} = i \end{aligned}$$

1. 9 2. -5 3. $4i$ 4. $10i$
5. $\sqrt{-25}$ 6. $\sqrt{-29}$ $\boxed{7.}$ $4 + 2\sqrt{-4}$ 8. $-3 - \sqrt{-10}$

Add or subtract as indicated. Write the answer in standard form $a + bi$ or $a - bi$. See example 10–4 B.

Example $(4 - 2i) - (6 + 5i)$

Solution
$$\begin{aligned} &= 4 - 2i - 6 - 5i && \text{Definition of subtration} \\ &= (4 - 6) + (-2i - 5i) && \text{Combine like terms} \\ &= -2 + (-7i) \\ &= -2 - 7i && \text{Definition of subtraction} \end{aligned}$$

9. $(1 + 2i) + (3 - i)$ 10. $(5 + 4i) + (3 + 5i)$ 11. $(5 - i) - (4 + 3i)$
12. $(1 - 5i) - (2 - i)$ 13. $(3 + \sqrt{-1}) + (2 - 3\sqrt{-1})$ $\boxed{14.}$ $(1 - \sqrt{-4}) - (3 + \sqrt{-9})$
15. $(-5 - \sqrt{-7}) - (4 + \sqrt{-7})$ 16. $(2 + \sqrt{-11}) + (3 - \sqrt{-11})$

Find the indicated products. Write the answer in standard form. See example 10–4 C.

Example $(2 - 3i)^2$

Solution
$$\begin{aligned} &= (2 - 3i)(2 - 3i) \\ &= 2(2 - 3i) - 3i(2 - 3i) && \text{Distributive property} \\ &= 4 - 6i - 6i + 9i^2 \\ &= 4 - 12i + 9(-1) && \text{Combine similar terms} \\ &= 4 - 12i - 9 && i^2 = -1 \\ &= -5 - 12i \end{aligned}$$

17. $3i(2 + 4i)$ 18. $4i(5 - 2i)$ 19. $(3 + 2i)(4 + i)$ 20. $(5 - i)(3 + 4i)$
21. $(5 - 4i)(5 + 4i)$ 22. $(5 - 5i)(5 + 5i)$ $\boxed{23.}$ $(4 + 7i)^2$ 24. $(3 - 2i)^2$

Rationalize the denominator. Write the answer in standard form $a + bi$ or $a - bi$. See example 10–4 D.

Example $\dfrac{3 - 2i}{1 + i}$

Solution

$$= \frac{(3 - 2i)(1 - i)}{(1 + i)(1 - i)}$$ Multiply numerator and denominator by conjugate of $1 + i$

$$= \frac{3 - 5i + 2i^2}{1^2 + 1^2}$$ Multiply as indicated

$$= \frac{3 - 5i - 2}{2}$$ $2i^2 = 2(-1) = -2$

$$= \frac{1 - 5i}{2}$$

$$= \frac{1}{2} - \frac{5}{2}i$$ Divide each term of numerator by 2

25. $\dfrac{5i}{2 + 3i}$ **26.** $\dfrac{6i}{6 - 7i}$ **27.** $\dfrac{4 + 2i}{3 - 5i}$ $\boxed{\textbf{28.}}$ $\dfrac{1 + i}{2 - i}$ **29.** $\dfrac{5 - i}{4 + 3i}$ **30.** $\dfrac{4 - 7i}{5 + 2i}$

Find the solution set of the following quadratic equations. See example 10–4 E.

Example $2y^2 - y = -5$

Solution $2y^2 - y + 5 = 0$ Write in standard form

$a = 2, b = -1, c = 5$

Using quadratic formula,

$$y = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(5)}}{2(1)}$$ Replace a with 2, b with -1, and c with 5

$$= \frac{1 \pm \sqrt{1 - 40}}{2}$$

$$= \frac{1 \pm \sqrt{-39}}{2}$$

$$= \frac{1 \pm i\sqrt{39}}{2}$$ $\sqrt{-39} = \sqrt{39} \cdot \sqrt{-1} = \sqrt{39} \cdot i = i\sqrt{39}$

The solution set is $\left\{\dfrac{1 - i\sqrt{39}}{2}, \dfrac{1 + i\sqrt{39}}{2}\right\}$.

31. $(x + 2)^2 = -16$ **32.** $(x - 5)^2 = -3$ **33.** $x^2 + x + 2 = 0$

34. $x^2 - 3x + 7 = 0$ **35.** $x^2 - 3x = -5$ **36.** $x^2 + 5x = -9$

37. $2y^2 + y + 4 = 0$ $\boxed{\textbf{38.}}$ $3y^2 - 2y + 3 = 0$ **39.** $(x + 3)(x - 2) = -11$

40. $(2y - 1)(3y - 2) = -3$

Determine the type of solution(s) that the following quadratic equations yield, using the discriminant. See example 10–4 F.

Example $3x^2 + 2x = 4$

Solution $3x^2 + 2x - 4 = 0$ — Write in standard form

$a = 3, b = 2, c = -4$

$$\begin{aligned} b^2 - 4ac &= (2)^2 - 4(3)(-4) \\ &= 4 + 48 \\ &= 52 \end{aligned}$$

Replace a with 3, b with 2, and c with -4

The two solutions are distinct and irrational since $52 > 0$ and 52 is *not* a perfect square.

41. $y^2 + 2y - 5 = 0$
42. $2x^2 + x + 3 = 0$
43. $4x^2 - 12x + 9 = 0$
44. $3y^2 + y - 1 = 0$
45. $9x^2 - 3x = 0$
46. $3y^2 + 5y + 2 = 0$
47. $(x + 4)(x + 3) = 1$
48. $(2x + 3)(x + 5) = -3$

Review exercises

Graph the following equations by finding *three* ordered pair solutions and then graphing the points. See section 7–2.

1. $y = 4x - 3$
2. $y = 2 - 3x$
3. Graph the system of equations
   $2x - y = 1$
   $x + y = 3$
   and find the solution. See section 8–1.
4. Find the equation of the line through the points $(1,-3)$ and $(4,5)$. Write the answer in standard form. See section 7–4.

Perform the indicated operation. See section 6–1.

5. $\dfrac{3x}{x-2} \cdot \dfrac{x^2 - x - 2}{4x^2}$

10–5 ■ The graphs of quadratic equations in two variables—quadratic functions

In chapter 7, we graphed *linear* equations in two variables such as

$$2x - 3y = 12 \text{ and } y = 4x - 5$$

The graphs of such equations were straight lines. In this section, we graph **quadratic equations in two variables** of the form

$$y = ax^2 + bx + c \quad (a \neq 0)$$

For example,

$$y = x^2 + 2x - 1$$

Values of x are arbitrarily chosen and corresponding values of y are found by substituting the value for x into the equation. Thus, when

a. $x = 1, y = (1)^2 + 2(1) - 1 = 1 + 2 - 1 = 2; (1,2)$
b. $x = -3, y = (-3)^2 + 2(-3) - 1 = 9 - 6 - 1 = 2; (-3,2)$
c. $x = 0, y = (0)^2 + 2(0) - 1 = -1; (0,-1)$

Thus, the ordered pairs $(1,2)$, $(-3,2)$, and $(0,-1)$ are solutions of the equation $y = x^2 + 2x - 1$.

Inspection will show us that for any chosen value of x, we get a unique (only one) value of y. Thus, *any quadratic equation in two variables of the form $y = ax^2 + bx + c$ does define a function.* Recall that we used the symbol $f(x)$ to replace y when defining a function. Thus, the **quadratic function** may be defined by

$$f(x) = ax^2 + bx + c, a \neq 0$$

Note $y = ax^2 + bx + c$ and $f(x) = ax^2 + bx + c$ define *the same function.* The latter gives the function a specific name, in this case f.

For example, the quadratic equations

$$y = x^2 + 2x + 1, \quad y = x^2 - 4x - 12, \quad \text{and} \quad y = 4x^2 - 3$$

can be used to define the quadratic functions f, g, and h such that

$$f(x) = x^2 + 2x + 1, \quad g(x) = x^2 - 4x - 12, \quad \text{and} \quad h(x) = 4x^2 - 3$$

As functions, the quadratic functions f, g, and h each have a domain and a range. *The domain of every quadratic function of the form $f(x) = ax^2 + bx + c$ is the set of real numbers* because we can evaluate the function for any real number we choose.

■ **Example 10–5 A**

1. Given $f(x) = x^2 + 2x + 1$, find $f(3)$.

$$\begin{aligned} f(3) &= (3)^2 + 2(3) + 1 \\ &= 9 + 6 + 1 \\ &= 16 \end{aligned}$$ Replace x with 3

Therefore, $f(3) = 16$ and the ordered pair $(3,16)$ is an element of the function f.

2. Given $g(x) = x^2 - 4x - 12$, find $g(6)$.

$$\begin{aligned} g(6) &= (6)^2 - 4(6) - 12 \\ &= 36 - 24 - 12 \\ &= 0 \end{aligned}$$ Replace x with 6

Therefore, $g(6) = 0$ and the ordered pair $(6,0)$ is an element of the function g.

3. Given $h(x) = 4x^2 - 3$, find $h\left(\frac{1}{2}\right)$.

$$\begin{aligned} h\left(\frac{1}{2}\right) &= 4\left(\frac{1}{2}\right)^2 - 3 \\ &= 1 - 3 \\ &= -2 \end{aligned}$$ Replace x with $\frac{1}{2}$

Therefore, $h\left(\frac{1}{2}\right) = -2$ and the ordered pair $\left(\frac{1}{2}, -2\right)$ is an element of function h.

Remember a function is *always* a set of ordered pairs.

▶ ***Quick check*** Given $f(x) = 2x^2 - 3x + 1$, find $f(-3)$, $f(0)$, and $f(1)$. State the answers as ordered pairs.

The parabola

Recall that the graph of a linear (first-degree) equation in two variables, or linear function, is a straight line. The graph of a quadratic (second-degree) equation in two variables, or quadratic function, is *not* a straight line. For this reason, we will require a number of points to plot the graph. The same procedure we used to graph linear equations can be applied to graph quadratic equations. The graph of any equation of the form $f(x) = y = ax^2 + bx + c$, $a \neq 0$ is, in fact, a special *curve,* called a **parabola.**

Consider the quadratic equation given by $f(x) = y = x^2 + 2x - 8$. Since we do not know what the graph looks like, we will need to choose a sufficient number of points.

x	*f(x)* or *y*	*(x,y)*
-5	$y = (-5)^2 + 2(-5) - 8 = 25 + (-10) - 8 = 7$	$(-5,7)$
-4	$y = (-4)^2 + 2(-4) - 8 = 16 + (-8) - 8 = 0$	$(-4,0)$
-3	$y = (-3)^2 + 2(-3) - 8 = 9 + (-6) - 8 = -5$	$(-3,-5)$
-2	$y = (-2)^2 + 2(-2) - 8 = 4 + (-4) - 8 = -8$	$(-2,-8)$
-1	$y = (-1)^2 + 2(-1) - 8 = 1 + (-2) - 8 = -9$	$(-1,-9)$
0	$y = (0)^2 + 2(0) - 8 = 0 + 0 - 8 = -8$	$(0,-8)$
1	$y = (1)^2 + 2(1) - 8 = 1 + 2 - 8 = -5$	$(1,-5)$
2	$y = (2)^2 + 2(2) - 8 = 4 + 4 - 8 = 0$	$(2,0)$
3	$y = (3)^2 + 2(3) - 8 = 9 + 6 - 8 = 7$	$(3,7)$

Plotting all the points and passing a *smooth curve* through them, we have the graph of the quadratic equation $f(x) = y = x^2 + 2x - 8$ (figure 10–1).

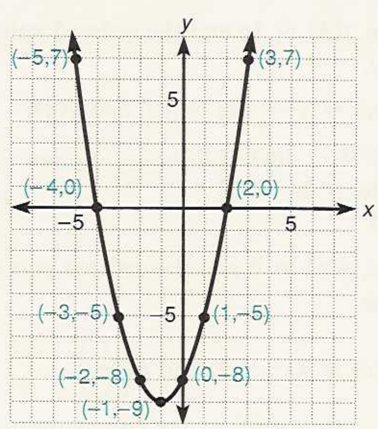


Figure 10–1

Note We *do not* connect the points with straight lines.

The x- and y-intercepts of a parabola

No matter how many points we plot, we cannot be sure that connecting all the points with a smooth curve will produce the correct graph and reveal all the important features of a curve. There are certain features of a parabola that we always wish to include in our graph, namely the x- and y-intercepts and the extreme (lowest or highest) point of the graph, called the *vertex*.

We observe from our example that when the curve crosses the x-axis, the value of y is zero, and when the curve crosses the y-axis, the value of x is zero. This is the same observation that we made with the linear equation, and we can generalize finding the x- and y-intercepts for any graph as follows:

1. To find the x-intercept(s), if there are any, we let $y = 0$ in the equation and solve for x.
2. To find the y-intercept, we let $x = 0$ in the equation and solve for y.

Example 10–5 B

Find the x- and y-intercepts.

1. $y = x^2 - 4$

a. Let $y = 0$

$(0) = x^2 - 4$ Replace y with 0

$0 = (x - 2)(x + 2)$ Factor right member, set each factor equal to 0 and solve

$x = 2$ or -2

The x-intercepts are 2 and -2. [The points are (2,0) and (-2,0).]

b. Let $x = 0$

$y = (0)^2 - 4$ Replace x with 0

$y = -4$

Therefore, the y-intercept is -4. [The point is (0,-4).]

2. $y = x^2 - 6x + 9$

a. Let $y = 0$

$(0) = x^2 - 6x + 9$ Replace y with 0

$0 = (x - 3)(x - 3)$ Factor, set each factor equal to 0 and solve

$x = 3$

Therefore, the x-intercept is 3. [The point is (3,0).]

b. Let $x = 0$

$y = (0)^2 - 6(0) + 9$ Replace x with 0

$y = 9$

Hence, the y-intercept is 9. [The point is (0,9).]

3. $y = -x^2 + 2x + 3$

a. Let $y = 0$

$(0) = -x^2 + 2x + 3$

$0 = x^2 - 2x - 3$ Multiply each member by -1

$0 = (x - 3)(x + 1)$ Factor right member and set each factor equal to 0

$x = 3$ or $x = -1$ Solve each equation

The x-intercepts are 3 and -1. [The points are (3,0) and (-1,0).]

b. Let $x = 0$

$$y = -(0)^2 + 2(0) + 3$$
$$= 3$$

The y-intercept is 3. [The point is (0,3).]

4. $y = x^2 + 1$

a. Let $y = 0$

$$(0) = x^2 + 1$$

Then $x^2 = -1$ and $x = \pm\sqrt{-1}$. Since $\sqrt{-1}$ is not a real number, there are no real solutions for x. Hence, the graph has no x-intercepts.

b. Let $x = 0$

$$y = (0)^2 + 1$$
$$= 1$$

The y-intercept is 1. [The point is (0,1).]

Note From these examples, we see that the y-intercept is always the constant c. If the quadratic equation is in standard form, the y-intercept will be the point $(0,c)$.

▶ ***Quick check*** Find the x- and y-intercepts. $y = 2x^2 - 3x + 1$

The vertex and the axis of symmetry of a parabola

We wish to find one remaining point of interest on the graph—the vertex. The vertex is the extreme point on the graph, that is, either the maximum or minimum value that the graph will attain. If our equation is in standard form, $y = ax^2 + bx + c$, we can show, but will not do so here, the x-component of the vertex is given by $x = \dfrac{-b}{2a}$. Once we have determined the x-component, we replace x with this value in our original function and generate the corresponding y-value. Recall our original example: $y = x^2 + 2x - 8$. The value of a is 1 and b is 2. Therefore

$$x = \frac{-b}{2a} = \frac{-2}{2(1)} = \frac{-2}{2} = -1$$

We then substitute this value for x in our original equation and we obtain

$$y = (-1)^2 + 2(-1) - 8 = 1 + (-2) - 8 = -9$$

Hence, in this case, the vertex is the point with coordinates $(-1,-9)$. This means no matter what value x takes, y, or $f(x)$, is *never* less than -9.

Note When the value of a, the coefficient of the squared term, is positive (as in this case), the parabola opens *upward* and the vertex is the *lowest* point of the graph. When a is negative, the parabola opens *downward* and the vertex is the *highest* point of the graph.

Example 10–5 C

Find the coordinates of the vertex of each parabola.

1. $y = x^2 - 4$

$y = x^2 + 0x - 4$ Write equation in standard form

Since $a = 1$ and $b = 0$, the vertex has coordinates

$$x = -\frac{b}{2a} = -\frac{(0)}{2(1)} = 0$$ Replace a with 1 and b with 0

$$y = (0)^2 - 4 = -4$$ Replace x with 0 in $y = x^2 - 4$

The vertex is the point $(0, -4)$.

2. $y = x^2 - 6x + 9$

Since $a = 1$ and $b = -6$, the vertex has coordinates

$$x = \frac{-b}{2a} = -\frac{(-6)}{2(1)} = \frac{6}{2} = 3$$

$$\begin{aligned}\text{and } y &= (3)^2 - 6(3) + 9\\ &= 9 - 18 + 9\\ &= 0\end{aligned}$$

The vertex is the point $(3,0)$.

3. $y = -x^2 + 2x + 3$

Since $a = -1$ and $b = 2$, so the vertex has coordinates

$$x = -\frac{b}{2a} = -\frac{(2)}{2(-1)} = \frac{2}{2} = 1$$

$$\begin{aligned}\text{and } y &= -(1)^2 + 2(1) + 3\\ &= -1 + 2 + 3\\ &= 4\end{aligned}$$

The vertex is the point $(1,4)$.

4. $y = x^2 + 1$

Since $a = 1$ and $b = 0$, so the vertex has coordinates

$$x = -\frac{b}{2a} = -\frac{(0)}{2(1)} = -\frac{0}{2} = 0$$

$$\begin{aligned}\text{and } y &= (0)^2 + 1\\ &= 1\end{aligned}$$

The vertex is the point $(0,1)$.

▶ ***Quick check*** Find the vertex. $y = 2x^2 - 3x + 1$

If the vertex is the point, (h,k), the vertical line, $x = h$, which passes through the vertex, is called the *axis of symmetry* in the graph of the parabola. The parabola is a symmetric curve and if we fold the graph along the axis of symmetry, the left half of the curve will coincide with the right half of the curve. For this reason, we choose two values of x that are greater than h and two values of x that are less than h when finding our arbitrary points to graph.

The graph of a quadratic equation

To draw a reasonably accurate graph of any quadratic equation in two variables, we take the following steps.

Graphing a quadratic equation in two variables

1. Find the coordinates of the x- and y-intercepts.
   a. Let $x = 0$, solve for the y-intercept.
   b. Let $y = 0$, solve for the x-intercept(s).
2. Find the coordinates of the vertex.
   a. $x = -\dfrac{b}{2a}$
   b. Replace x with the value of $-\dfrac{b}{2a}$ in the original equation and solve for y.
3. Find the coordinates of four arbitrarily chosen points. Choose values of x such that, if the point (h,k) is the vertex of the parabola,
   a. two values are less than h and
   b. two values are greater than h.
4. Draw a smooth curve through the resulting points.

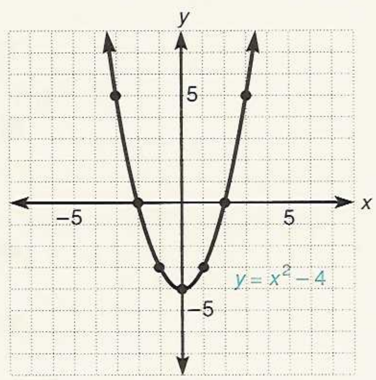
Example 10–5 D

Graph the following quadratic equations. Determine the equation of the axis of symmetry.

1. $y = x^2 - 4$
In our previous examples, we found the x- and y-intercepts and the vertex. We need only determine four more points and we will be ready to graph the function.

x	y	
2	0	x-intercepts
−2	0	
0	−4	y-intercept and vertex
−1	−3	Arbitrary points
1	−3	
−3	5	
3	5	

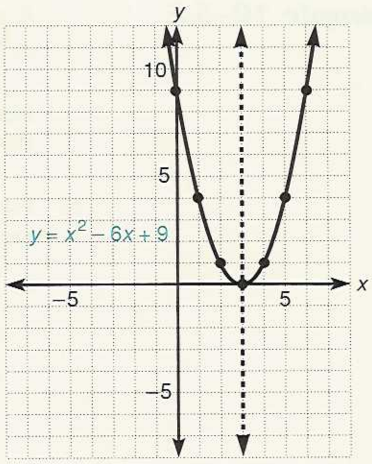
The axis of symmetry is the line $x = 0$ (y-axis).



2. $y = x^2 - 6x + 9$

x	y	
3	0	x-intercept and vertex
0	9	y-intercept
1	4	Arbitrary points
2	1	
4	1	
5	4	
6	9	

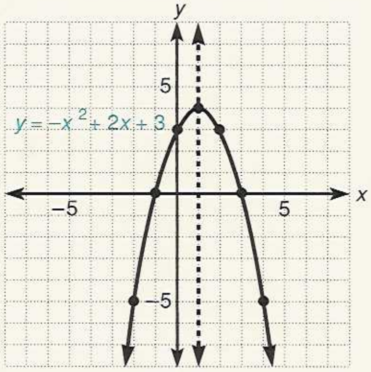
The axis of symmetry is the line $x = 3$.



3. $y = -x^2 + 2x + 3$

x	y	
3	0	x-intercepts
-1	0	
0	3	y-intercept
1	4	vertex
-2	-5	Arbitrary points
2	3	
4	-5	

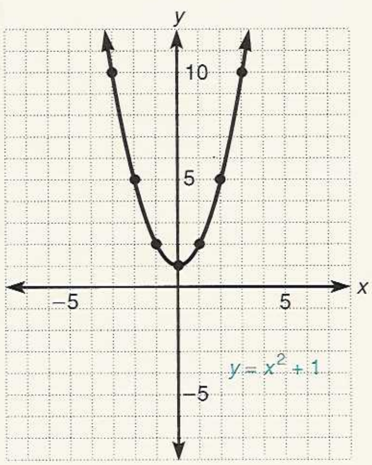
The axis of symmetry is the line $x = 1$.



4. $y = x^2 + 1$

x	y	
0	1	y-intercept and vertex
-3	10	Arbitrary points
-2	5	
-1	2	
1	2	
2	5	
3	10	

The axis of symmetry is the line $x = 0$ (y-axis).



▶ ***Quick check*** Graph the equation and determine the equation of the axis of symmetry. $y = 2x^2 - 3x + 1$ ■

Quadratic equations are used in many physical situations. For example, if an object is thrown into the air, the graph of the distance the object travels versus the time it travels is a parabola.

Example 10–5 E

A projectile is fired vertically into the air. Its distance s in feet above the ground in t seconds is given by $s = 160t - 16t^2$.

Note s is defined as a quadratic function of t. That is, the distance the object travels *changes with* time.

Find the highest point of the projectile (the vertex of the parabola) and the moment when the projectile will strike the ground. Graph the equation.

a. The vertex is the highest point since the parabola opens downward.

$$s = -16t^2 + 160t$$

where $a = -16$, $b = 160$. The t value of the vertex is

$$t = -\frac{b}{2a} = -\frac{(160)}{2(-16)} = -\frac{160}{-32} = 5 \qquad \text{Replace } a \text{ with } -16 \text{ and } b \text{ with } 160$$

The height will be

$$s = -16(5)^2 + 160(5) = -400 + 800 = 400 \text{ feet}$$

The maximum height, $s = 400$ feet, is attained when $t = 5$ seconds.

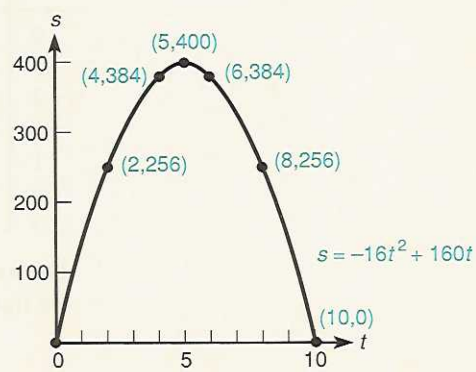
b. The projectile will strike the ground when $s = 0$. Therefore, we set $s = 0$ and solve for t.

$$\begin{aligned} 0 &= -16t^2 + 160t && \text{Replace } s \text{ with } 0 \\ &= -16t(t - 10) && \text{Set each factor equal to 0 and solve} \\ t &= 0 \text{ or } 10 \end{aligned}$$

The value $t = 0$ seconds represents when the projectile was fired. Hence, the value $t = 10$ seconds represents the time when the projectile will strike the ground.

c. To graph the equation, we plot time, t, along the horizontal axis and distance, s, along the vertical axis.

t	s	
5	400	vertex
0	0	t- and s-intercept
10	0	t-intercept
2	256	Arbitrary points
4	384	
6	384	
8	256	



Note We have used different scales on the t- and s-axes, and we do not represent values on the graph for negative time or distance, since they do not have any meaning in this example.

Mastery points

Can you

- Evaluate a quadratic function?
- Find the x- and y-intercepts of the graph of a quadratic equation?
- Find the coordinates of the vertex of the graph of a quadratic equation?
- Graph a quadratic function in two variables?
- Determine the equation of the axis of symmetry?

Exercise 10–5

Find the value of the following quadratic functions at the given values of x. State the answer as ordered pairs. See example 10–5 A.

Example Given $f(x) = 2x^2 - 3x + 1$, find $f(-3), f(0), f(1)$.

Solution $f(-3) = 2(-3)^2 - 3(-3) + 1 = 18 + 9 + 1 = 28;$ $(-3,28)$
$f(0) = 2(0)^2 - 3(0) + 1 = 0 - 0 + 1 = 1;$ $(0,1)$
$f(1) = 2(1)^2 - 3(1) + 1 = 2 - 3 + 1 = 0;$ $(1,0)$

1. $f(x) = x^2 + 3x - 4; f(-1), f(0), f(3)$

2. $g(x) = x^2 - x - 1; g(-2), g(0), g(1)$

3. $h(x) = 4x^2 + x + 5; h(-3), h(0), h(2)$

4. $f(x) = 3x^2 - 4x + 7; f(-4), f(0), f\left(\frac{1}{3}\right)$

5. $f(x) = 5x^2 - 3x; f(-6), f(0), f(6)$

6. $g(x) = 8x - 2x^2; g(-2), g(0), g\left(\frac{1}{2}\right)$

7. $h(x) = 5x^2 - 1; h(-3), h(0), h(5)$

8. $f(x) = 4x^2 + 2; f\left(\frac{-1}{2}\right), f(0), f(8)$

9. $g(x) = 4 - 2x - 3x^2; g(-2), g(0), g\left(\frac{3}{4}\right)$

10. $h(x) = 10 + 7x - 2x^2; h(-4), h(0), h(1.2)$

Find the x- and y-intercepts. If they do not exist, so state. See example 10–5 B.

Example $y = 2x^2 - 3x + 1$

Solution a. Let $y = 0$

$(0) = 2x^2 - 3x + 1$ Replace y with 0
$0 = (2x - 1)(x - 1)$ Factor right member
$2x - 1 = 0$ or $x - 1 = 0$ Set each factor equal to 0 and solve
$x = \frac{1}{2}$ $x = 1$

The x-intercepts are $\frac{1}{2}$ and 1. $\left[\text{The points are } \left(\frac{1}{2},0\right) \text{ and } (1,0).\right]$

b. Let $x = 0$
Then $y = 1$. The y-intercept is 1. [The point is (0,1).]

11. $y = x^2 - 16$

12. $y = x^2 - 9$

13. $y = x^2 - 6x + 8$

14. $y = x^2 + 11x - 12$

15. $y = x^2 + 8x + 12$

16. $y = x^2 - 4x - 12$

17. $y = 5 - x^2$
18. $y = 7 - x^2$
19. $y = x^2 + 6x + 9$
20. $y = x^2 - 4x + 4$
21. $y = x^2 + 5$
22. $y = x^2 + 6$
23. $y = x^2 + 4x + 6$
24. $y = x^2 + 2x + 5$
25. $y = -x^2 + 8x - 16$
26. $y = 25 - x^2$
27. $y = 2x^2 + 3x + 1$
28. $y = 2x^2 - 7x + 6$
29. $y = -2x^2 - x + 6$
30. $y = -3x^2 + 7x + 6$

Find the vertex. Find the equation of the axis of symmetry. See example 10–5 C.

Example $y = 2x^2 - 3x + 1$

Solution Now $a = 2$, $b = -3$, and $c = 1$

so $x = -\frac{b}{2a} = -\frac{(-3)}{2(2)} = \frac{3}{4}$ Replace a with 2 and b with -3

then $y = 2\left(\frac{3}{4}\right)^2 - 3\left(\frac{3}{4}\right) + 1$ Replace x with $\frac{3}{4}$

$$= 2\left(\frac{9}{16}\right) - \frac{9}{4} + 1$$
$$= \frac{9}{8} - \frac{9}{4} + 1$$
$$= \frac{9}{8} - \frac{18}{8} + \frac{8}{8}$$
$$= -\frac{1}{8}$$

The vertex is the point $\left(\frac{3}{4}, -\frac{1}{8}\right)$. The axis of symmetry is $x = \frac{3}{4}$.

31. $y = x^2 - 16$
32. $y = x^2 - 9$
33. $y = x^2 - 6x + 8$
34. $y = x^2 + 11x - 12$
35. $y = x^2 + 8x + 12$
36. $y = x^2 - 4x - 12$
37. $y = 5 - x^2$
38. $y = 7 - x^2$
39. $y = x^2 + 6x + 9$
40. $y = x^2 - 4x + 4$
41. $y = x^2 + 5$
42. $y = x^2 + 6$
43. $y = x^2 + 4x + 6$
44. $y = x^2 + 2x + 5$
45. $y = -x^2 + 8x - 16$
46. $y = 25 - x^2$
47. $y = 2x^2 + 3x + 1$
48. $y = 2x^2 - 7x + 6$
49. $y = -2x^2 - x + 6$
50. $y = -3x^2 + 7x + 6$

Graph the following equations. Include the vertex and the points at which the graph crosses each axis. See example 10–5 D.

Example $y = 2x^2 - 3x + 1$

Note $a = 2$, which is positive so the parabola opens up.

Solution The vertex is at $\left(\frac{3}{4}, -\frac{1}{8}\right)$.

Let $x = 0$. Then

$y = 2(0) - 3(0) + 1 = 1$

so the y-intercept is at $(0,1)$.

Let $y = 0$. Then

$$(0) = 2x^2 - 3x + 1 = (2x - 1)(x - 1)$$

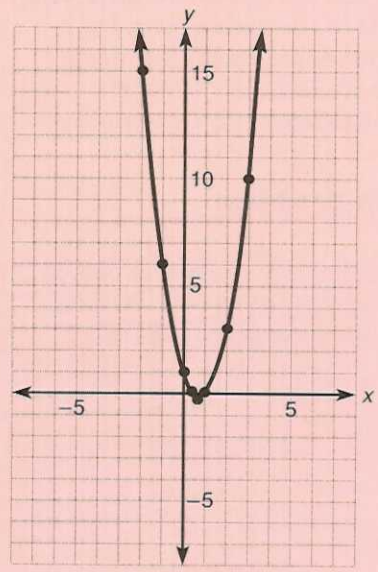
so $2x - 1 = 0$ or $x - 1 = 0$

$2x = 1$ $\quad$ $x = 1$

$x = \frac{1}{2}$ $\quad$ $x = 1$

The x-intercepts are at $\left(\frac{1}{2}, 0\right)$ and $(1,0)$.

x	y	
0	1	y-intercept
1	0	x-intercepts
$\frac{1}{2}$	0	
$\frac{3}{4}$	$-\frac{1}{8}$	Vertex
2	3	Arbitrary points
-1	6	
3	10	
-2	15	



51. $y = x^2 - 16$

52. $y = x^2 - 9$

53. $y = x^2 - 6x + 8$

54. $y = x^2 + 11x - 12$

55. $y = x^2 + 8x + 12$

56. $y = x^2 - 4x - 12$

57. $y = 5 - x^2$

58. $y = 7 - x^2$

59. $y = x^2 - 4x + 4$

60. $y = x^2 + 6x + 9$

61. $y = x^2 + 6$

62. $y = x^2 + 5$

63. $y = x^2 + x - 6$

64. $y = x^2 + 2x + 5$

65. $y = -x^2 + 4x - 3$

66. $y = 25 - x^2$

67. $y = 2x^2 + 3x + 1$

68. $y = 2x^2 - 7x + 6$

69. $y = -2x^2 - x + 6$

70. $y = -3x^2 + 7x + 6$

Graph each equation by plotting the variable for which the equation is solved along the vertical axis and by plotting the other variable along the horizontal axis. Graph the equation only in the regions for which the equation would have meaning. See example 10–5 E.

71. When a ball rolls down an inclined plane, it travels a distance $d = 6t + \frac{t^2}{2}$ feet in t seconds. Plot the graph showing how d depends on t. How long will it take the ball to travel 14 feet?

72. The output power P of a 100-volt electric generator is defined by $P = 100I - 5I^2$, where I is in amperes. Plot the graph showing how P depends on I.

73. The current in a circuit flows according to the equation $i = 12 - 12t^2$, where i is the current and t is the time in seconds. Plot the graph of the relation given by the equation labeling the horizontal axis the t-axis.

74. If a projectile is fired vertically into the air with an initial velocity of 80 feet per second, the distance in feet above the ground in t seconds is given by $s = 80t - 16t^2$. Find the projectile's maximum height and when the projectile will strike the ground. Plot the graph showing how s depends on t.

75. Referring to exercise 74, if the initial velocity is 96 ft/sec, the equation is $s = 96t - 16t^2$. Find the maximum height and when the projectile will strike the ground. Plot the graph of this equation using the t-axis as the horizontal axis.

76. The distance s through which an object will fall in t seconds is $s = 16t^2$. Plot the graph showing the relation between s and t for the first 5 seconds.

77. An object is dropped from the top of the Empire State Building (1,250 feet tall), and the distance that the object is from the ground is given by the equation $s = 1{,}250 - 16t^2$. Plot the graph showing how s depends on t and determine when the object will strike the ground. (t is time in seconds.)

Chapter 10 lead-in problem

A rock is dropped from the top of the Washington Monument. If the monument is 555 feet tall, how long will it take the rock to strike the ground?

Solution

We use $s = 16t^2$, where s is the distance the rock fell and t is the time in seconds.

$555 = 16t^2$ Replace s with 555

$t^2 = \frac{555}{16}$ Divide each member by 16

$t = \sqrt{\frac{555}{16}} = \frac{\sqrt{555}}{4} \approx 5.9$ Extract the roots

or $t = -\sqrt{\frac{555}{16}} = -\frac{\sqrt{555}}{4} \approx -5.9$

Reject the negative value since time t must be positive. Thus, the rock will strike the ground in approximately 5.9 seconds.

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Chapter 10 summary

1. We can solve equations of the form $x^2 = k$ and $(x + q)^2$ or $(px + q)^2 = k$, $k \geq 0$, by **extracting the roots.**
2. If $x^2 = k$, then $x = \pm\sqrt{k}$, and if $(x + q)^2 = k$, then $x + q = \pm\sqrt{k}$ and $x = -q \pm \sqrt{k}$.
3. Any quadratic equation can be solved by **completing the square.**
4. Given the quadratic equation $ax^2 + bx + c = 0$, the **quadratic formula** states $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.
5. A **quadratic function** is defined by $f(x) = ax^2 + bx + c$, where a, b, and c are constants, $a \neq 0$.
6. A **complex number** is any number that can be written in the form $a + bi$, where a and b are real numbers and i equals $\sqrt{-1}$.
7. The graph of a quadratic function is a **parabola.**
8. The coordinates of the vertex of the graph of a quadratic function are found by $x = -\dfrac{b}{2a}$ and replacing x with $-\dfrac{b}{2a}$ to find y.
9. In the quadratic equation in two variables, $y = ax^2 + bx + c$, the graph of the parabola
   a. opens up if $a > 0$
   b. opens down if $a < 0$
10. The equation of the *axis of symmetry* is $x = h$, where the vertex is the point (h,k).

Chapter 10 error analysis

1. Solving quadratic equations by completing the square
   Example: Find the solution set of $x^2 + x - 8 = 0$ by completing the square.
   $$x^2 + x = 8$$
   $$x^2 + x + \frac{1}{4} = 8$$
   $$\left(x + \frac{1}{2}\right)^2 = 8$$
   $$x + \frac{1}{2} = \pm\sqrt{8} = \pm 2\sqrt{2}$$
   $$x = -\frac{1}{2} \pm 2\sqrt{2} = \frac{-1 \pm 4\sqrt{2}}{2} \quad \left\{\frac{-1 - 4\sqrt{2}}{2}, \frac{-1 + 4\sqrt{2}}{2}\right\}$$
   Correct answer: $\left\{\dfrac{-1 - \sqrt{33}}{2}, \dfrac{-1 + \sqrt{33}}{2}\right\}$
   What error was made? (*see page 411*)
2. Solving quadratic equations by quadratic formula
   Example: Find the solution set of $x^2 + 2x - 5 = 0$ by quadratic formula.
   $a = 1, b = 2, c = -5$
   $$x = -2 \pm \frac{\sqrt{(2)^2 - 4(1)(-5)}}{2(1)}$$
   $$= -2 \pm \frac{\sqrt{24}}{2} = -2 \pm \frac{2\sqrt{6}}{2}$$
   $$= -2 \pm \sqrt{6} \quad \{-2 - \sqrt{6}, -2 + \sqrt{6}\}$$
   Correct answer:
   $\{-1 - \sqrt{6}, -1 + \sqrt{6}\}$
   What error was made? (*see page 418*)
3. Finding the solution set of a quadratic equation by quadratic formula
   Example: Find the solution set of $2x^2 - x = 4$ by quadratic formula.
   $a = 2, b = -1, c = 4$
   Correct answer: $a = 2, b = -1, c = -4$
   What error was made? (*see page 418*)
4. Extracting the roots
   Example: Find the solution set of $x^2 = 25$ by extracting roots.
   $x^2 = 25$
   $x = \sqrt{25} = 5 \quad \{5\}$
   Correct answer: $\{-5,5\}$
   What error was made? (*see page 405*)
5. Product of complex numbers
   Example: $(2 + 3i)(1 - i) = (2)(1) - 2i + 3i - 3i^2$
   $= 2 - 2i + 3i - 3$
   $= -1 + i$
   Correct answer: $5 + i$
   What error was made? (*see page 426*)
6. Finding the y-intercept of a parabola
   Example: Given the quadratic equation $y = 2x^2 - 2x - 3$, so the y-intercept is $(0,3)$.
   Correct answer: The y-intercept is $(0,-3)$.
   What error was made? (*see page 435*)

7. Graph of a parabola
Example: The parabola that is the graph of $y = 2x^2 + x - 3$ opens *downward.*
Correct answer: opens *upward*
What error was made? (*see page 436*)

8. Squaring a radical binomial
Example: $(\sqrt{6} + \sqrt{5})^2 = (\sqrt{6})^2 + (\sqrt{5})^2 = 6 + 5 = 11$
Correct answer: $11 + 2\sqrt{30}$
What error was made? (*see page 387*)

9. Multiplying square roots
Example: $\sqrt{-6} \cdot \sqrt{-6} = \sqrt{-6 \cdot -6} = \sqrt{36} = 6$
Correct answer: -6
What error was made? (*see page 426*)

10. Rationalizing the denominator of an nth root
Example: $\dfrac{3}{\sqrt[4]{xy^2}} = \dfrac{3}{\sqrt[4]{xy^2}} \cdot \dfrac{\sqrt[4]{xy^2}}{\sqrt[4]{xy^2}} = \dfrac{3\sqrt[4]{xy^2}}{xy^2}$
Correct answer: $\dfrac{3\sqrt[4]{x^3y^2}}{xy}$
What error was made? (*see page 381*)

Chapter 10 critical thinking

Given three consecutive integers, the product of the first and third is always one less than the square of the middle one.

Chapter 10 review

[10–1]

Find the solution set of the following quadratic equations by extracting the roots or by factoring.

1. $x^2 = 100$
2. $x^2 - 25 = 0$
3. $z^2 = 2$
4. $y^2 - 6 = 0$
5. $6x^2 = 24$
6. $8x^2 - 96 = 0$
7. $\dfrac{3}{4}x^2 = 12$
8. $\dfrac{2}{3}x^2 - 8 = 0$
9. $\dfrac{x^2}{3} - 8 = \dfrac{1}{4}$
10. $x^2 - x - 42 = 0$
11. $3y^2 - 7y + 4 = 0$

[10–2]

Find the solution set by completing the square.

12. $x^2 - 6x + 4 = 0$
13. $z^2 - 10z + 4 = 0$
14. $2a^2 - 8a = -1$
15. $3y^2 - 6y - 5 = 0$
16. $4 - x^2 = 5x$
17. $5 = 11y - y^2$
18. $x(4x - 1) = 3$
19. $(x - 2)(x + 1) = 1$
20. $4x^2 - 3 = 3x - 1$
21. $\dfrac{3}{5}x^2 + \dfrac{1}{5}x = 2$
22. The length of a rectangle is 2 meters more than three times the width. Its area is 16 square meters. What are its dimensions? (Solve by completing the square.)

[10–3]

Find the solution set using the quadratic formula.

23. $x^2 - 2x - 5 = 0$
24. $x^2 - 8 = -4x$
25. $2y^2 - 3y = 5$
26. $3a^2 = 8 - 7a$
27. $2x^2 - 9 = 0$
28. $4x^2 = -7x$
29. $x^2 - \dfrac{2}{3}x = \dfrac{4}{3}$
30. $2x + \dfrac{3}{4} = \dfrac{3}{2}x^2$
31. A metal bar is to be divided into two pieces so that one piece is 3 inches longer than the other. If the sum of the squares of the two lengths is 117 square inches, find the two lengths. (Use the quadratic formula.)

[10–4]

Perform the indicated operations on the given complex numbers. Write the answer in standard form $a + bi$ or $a - bi$.

32. $(3 - 5i) + (2 + 4i)$
33. $(7 + i) - (3 - 8i)$
34. $(6 + i)(2 - 3i)$
35. $(-3 + 9i)(2 - i)$
36. $(6 - 4i)(6 + 4i)$
37. $(5 - 3i)^2$

Rationalize the denominator. Write the answer in standard form $a + bi$ or $a - bi$.

38. $\frac{3i}{1+i}$ 39. $\frac{-4i}{2-i}$ 40. $\frac{2-i}{3-i}$ 41. $\frac{5+2i}{4+3i}$

Find the solution set of the following quadratic equations.

42. $x^2 + 4x + 7 = 0$ 43. $4y^2 - y = -5$ 44. $(x+1)(x-1) = -8$

45. $(2x+3)^2 = -4$

Determine the type of solutions the following quadratic equations will yield, using the discriminant.

46. $y^2 - 16y + 64 = 0$ 47. $3x^2 - x - 2 = 0$ 48. $5y^2 - 2y + 3 = 0$

49. $3x^2 + x - 3 = 0$

[10–5]

Evaluate each quadratic function at the given values of x.

50. $f(x) = x^2 - 3x - 5; f(-5), f(0), f(1)$ 51. $g(x) = 4 + 5x - 2x^2; g(-1), g(0), g(3)$

52. $h(x) = 4x^2 + 2x; h(-3), h(0), h(4)$ 53. $f(x) = 12 - 3x^2; f(-4), f(0), f(2)$

Find the x- and y-intercepts and the vertex in the graph of each quadratic equation. Find the equation of the axis of symmetry. Sketch the graph.

54. $y = x^2 - 4x - 12$ 55. $y = 5x^2 - 6x + 1$ 56. $y = 8 - 2x - x^2$ 57. $y = 2 + x - 3x^2$

58. $y = 5x^2 - 2x$ 59. $y = x - 3x^2$ 60. $y = 4x^2 - 8$ 61. $y = 9 - x^2$

62. $y = x^2 + 2$ 63. $y = x^2 + 2x + 3$

[10–2]

64. A particular projectile is distance d in feet from its starting point after t seconds of time has elapsed according to the formula $d = 2t^2 - 7t + 3$. How many seconds will it take to travel 12 feet?

65. The area of a tennis court is 2,800 sq. ft. Find the length of the court if the length is $3\frac{1}{9}$ times the width.

Final examination

[1–3] 1. Insert the proper inequality symbol, $<$ or $>$, to make the statement $|-5| \quad |4|$ true.

Perform the indicated operations and simplify the expression.

[1–8] 2. $38 - 10 \div 5 + 3 \cdot 4 - 2^3 + \sqrt{4}$

[1–8] 3. $-4[9 - 3(9-4) + 6]$

[2–2] 4. Evaluate the expression $a - b(2c - d)$ when $a = -4$, $b = 3$, $c = 5$, and $d = -6$.

Simplify the following and leave the answers with only positive exponents.

[3–4] 5. $x^5 \cdot x^{-3} \cdot x^4$ [3–4] 6. $\frac{2x^{-3}}{4x^2}$ [3–4] 7. $(3x^2y^3)(-4xy^2)$

[3–4] 8. $(3xy^{-2})^{-3}$ [3–4] 9. $(7x^3z^2)^0$

Remove the grouping symbols and combine.

[2–3] 10. $(4x^2 - y^2) - (2x^2 + y^2) + (5x^2 - 6y^2)$ [2–3] 11. $5x - (x - y) - 2x + y - (2x + 5y)$

Perform the indicated operations and simplify.

[3–2] 12. $(y+9)(y-9)$ [3–2] 13. $(7z - 3w)^2$ [3–2] 14. $(x+4)(5x^2 - 3x + 1)$

[5–3] 15. $\frac{5xy^2 - 3x^4y + x^2y^2}{xy}$ [5–3] 16. $(8y^2 - 2y - 3) \div (2y - 1)$

Find the solution set of the following equations.

[2–6] 17. $2(x + 1) - 3(x - 3) = 4$

[4–7] 18. $x^2 - 11x - 12 = 0$

[6–5] 19. $3 + \frac{2}{x^2} - \frac{7}{x} = 0$

[6–5] 20. $\frac{3x}{6} - 2 = \frac{5x}{4}$

[4–7] 21. $8x^2 = 12x$

Completely factor the following expressions.

[4–1] 22. $3x^2 - 6xy + 9x$

[4–2] 23. $a^2 - 4a - 21$

[4–3] 24. $4x^2 - 12x + 5$

[4–4] 25. $9a^2 - 64$

[4–6] 26. $6ax - 2ay + 3bx - by$

[4–4] 27. $x^2 - 10x + 25$

[4–8] 28. The product of two consecutive integers is 132. Find the integers.

Perform the indicated operations and reduce to lowest terms.

[6–1] 29. $\frac{x^2 + 7x + 6}{x^2 - 4} \cdot \frac{x - 2}{x + 6}$

[6–1] 30. $\frac{3x}{4x - 8} \div \frac{9x}{x^2 - 4x + 4}$

[6–2] 31. $\frac{9}{x - 6} - \frac{5}{6 - x}$

[6–2] 32. $\frac{x - 2}{x + 5} + \frac{x + 4}{x^2 - 25}$

[6–4] 33. Simplify the complex fraction $\dfrac{5 + \frac{4}{y}}{4 - \frac{6}{y}}$.

[5–4] 34. Find the value of x if $15 : 6 = 8 : x$.

[5–4] 35. What is the ratio of 42 oz to 5 lb?

[7–4] 36. Find the equation of the line passing through points $(-2,5)$ and $(1,-1)$. Write the answer in standard form.

[7–4] 37. Given the equation $2x - 3y = 9$, find the slope m and the y-intercept b of the line.

[10–5] 38. Given $f(x) = 3x^2 + 3x - 1$, find (a) $f(2)$, (b) $f(0)$, (c) $f(-1)$. Write the answers as ordered pairs.

[8–3] 39. Find the solution set of the system of equations
$x - 2y = 3$
$2x - 3y = -5$.

[2–8] 40. The perimeter of a rectangle is 34 feet. If the length is 2 more than twice the width, what are the dimensions of the rectangle?

Simplify the following expressions by performing the indicated operations. Rationalize all denominators.

[9–4] 41. $\sqrt{27} - \sqrt{48}$

[9–5] 42. $\sqrt{3}(\sqrt{2} + \sqrt{3})$

[9–5] 43. $(4 + \sqrt{3})(4 - \sqrt{3})$

[9–5] 44. $(2 - \sqrt{7})^2$

[9–3] 45. $\frac{3}{3 - \sqrt{5}}$

[9–1] 46. $\sqrt[3]{-27}$

[9–6] 47. $(16)^{3/4}$

[9–7] 48. Find the solution set of the equation $\sqrt{x + 1} - 1 = x$.

[7–4] 49. Sketch the graph of $3x + 2y = 12$ using the x- and y-intercepts.

[10–5] 50. Sketch the graph of $y = x^2 - 5x - 6$ using the vertex, x- and y-intercepts, and four arbitrary points.

[10–3] 51. Find the solution set of the quadratic equation $4x^2 - 7x = 3$ by any method.

Perform the indicated operations on the following complex numbers.

[10–4] 52. $(3 - 9i) - (2 + 10i)$

[10–4] 53. $(5 + 7i)(2 - i)$

[10–4] 54. $(8 - i)(8 + i)$

[10–4] 55. $(5 + 7i)^2$

[10–4] 56. $(3 - \sqrt{-4})(2 + \sqrt{-4})$

[10–4] 57. $(1 - \sqrt{-5})(1 + \sqrt{-5})$

Rationalize the denominator of the expression.

[10–4] 58. $\frac{5 - 3i}{6 + i}$

59. $\frac{3 + 4i}{2 - 3i}$

Check:
$$\sqrt{(3)+6} = (3)$$
$$\sqrt{9} = 3$$
$$3 = 3 \text{ (true)}$$
$$\sqrt{(-2)+6} = (-2)$$
$$\sqrt{4} = -2$$
$$2 = -2 \text{ (false)}$$
$\{3\}$

47.
$$\sqrt{x-4} = x-6$$
$$(\sqrt{x-4})^2 = (x-6)^2$$
$$x-4 = (x-6)(x-6)$$
$$x-4 = x^2-6x-6x+36$$
$$x-4 = x^2-12x+36$$
$$0 = x^2-13x+40$$
$$0 = (x-8)(x-5)$$
$$x-8 = 0 \text{ or } x-5 = 0$$
$$x = 8 \text{ or } \quad x = 5$$
Check:
$$\sqrt{(8)-4} = (8)-6$$
$$\sqrt{4} = 2$$
$$2 = 2 \text{ (true)}$$
$$\sqrt{(5)-4} = (5)-6$$
$$\sqrt{1} = -1$$
$$1 = -1 \text{ (false)}$$
$\{8\}$

57. Let $x =$ the number.
$$\sqrt{x+12} = x$$
$$(\sqrt{x+12})^2 = (x)^2$$
$$x+12 = x^2$$
$$0 = x^2-x-12$$
$$0 = (x-4)(x+3)$$
$$x-4 = 0 \text{ or } x+3 = 0$$
$$x = 4 \text{ or } \quad x = -3$$
Check:
$$\sqrt{(4)+12} = (4)$$
$$\sqrt{16} = 4$$
$$4 = 4 \text{ (true)}$$
$$\sqrt{(-3)+12} = (-3)$$
$$\sqrt{9} = -3$$
$$3 = -3 \text{ (false)}$$
Hence the number is 4.

Review exercises

1. $(x+2)(x-2)$ **2.** $(x+3)(x+6)$ **3.** $(x+2)(x-5)$ **4.** $(x-3)^2$ **5.** 9 **6.** 7 **7.** 11 **8.** $\{-8,8\}$

Chapter 9 review

1. 9 **2.** 5 **3.** -3 **4.** -7 **5.** $2\sqrt{10}$ **6.** $3ab\sqrt{2b}$ **7.** $2\sqrt{7}$ **8.** $6\sqrt{5}$ **9.** $\dfrac{4\sqrt{17}}{17}$ **10.** $\dfrac{\sqrt{14}}{6}$ **11.** $\dfrac{\sqrt{ab}}{b}$ **12.** $\dfrac{\sqrt{xy}}{y^2}$ **13.** $\dfrac{\sqrt{ab}}{b}$ **14.** $\dfrac{2\sqrt{xy}}{y}$ **15.** $7\sqrt{7}$ **16.** $8\sqrt{2}$ **17.** $3\sqrt{5}$ **18.** $24\sqrt{3}$ **19.** $\sqrt{2a}$ **20.** $17\sqrt{x}$ **21.** $\sqrt{15}-\sqrt{21}$ **22.** $2\sqrt{35}+2\sqrt{15}$ **23.** $8-2\sqrt{7}$ **24.** $39-12\sqrt{3}$ **25.** $8+2\sqrt{15}$ **26.** $4a-b$ **27.** $-\sqrt{3}-2$ **28.** $\dfrac{-\sqrt{6}+4}{5}$ **29.** $\dfrac{\sqrt{a}-b}{a-b^2}$ **30.** $\dfrac{\sqrt{xy}-x}{y-x}$ **31.** $\dfrac{a^2-a\sqrt{b}}{a^2-b}$ **32.** $\dfrac{-11-6\sqrt{2}}{7}$ **33.** 6 **34.** 4 **35.** -2 **36.** $\dfrac{1}{4}$ **37.** a **38.** $b^{13/12}$ **39.** $a^{1/4}$ **40.** $a^{9/8}$ **41.** $8a^3b^6$ **42.** $a^{3/2}b^{1/2}$ **43.** $\{64\}$ **44.** $\{53\}$ **45.** $\{4\}$ **46.** $\{1\}$ **47.** $\{2\}$ **48.** $\{3\}$ **49.** $\{-2\}$ **50.** $\{4,3\}$

Chapter 9 cumulative test

1. 45 **2.** x^7 **3.** x^6 **4.** $2x^2+2x+13$ **5.** $8a^4b^3-12a^3b^4+16a^2b^5$ **6.** $2a^3b^2$ **7.** -36 **8.** $9a^2-6ab+b^2$ **9.** $\dfrac{a-3}{3a+6}$ **10.** $16\sqrt{3}$ **11.** -2 **12.** $\dfrac{x+3}{x-1}$ **13.** $\dfrac{x\sqrt{x}+\sqrt{xy}}{x^2-y}$ **14.** $3xy^2\sqrt[3]{3xz}$ **15.** $25x^2-y^2$ **16.** $2x-4y$ **17.** $2a^3b^3(3b-b^2+4a^2)$ **18.** $(5c+d)(5c-d)$ **19.** $(2x-1)(x+4)$ **20.** $(y^2+2z)(y^2-2z)$ **21.** $(2x+1)(3x+4)$ **22.** $(x+7)(x-4)$ **23.** $\left\{-\dfrac{1}{3}\right\}$ **24.** $\{-3,3\}$ **25.** $\{-6\}$ **26.** $\{12\}$ **27.** $\left\{\dfrac{19}{12}\right\}$ **28.** $\left\{-1,-\dfrac{1}{2}\right\}$ **29.** $1<x<8$ **30.** $x>\dfrac{7}{2}$ **31.** -3 **32.** $y=4x-2$; slope is 4; y-intercept is $(0,-2)$ **33.** $\left(\dfrac{7}{8},\dfrac{3}{8}\right)$ **34.** 14, 56 **35.** 1,350 **36.** 16, 18 **37.** 21 feet by 27 feet

Chapter 10

Exercise 10–1

Answers to odd-numbered problems

1. $\{-5,3\}$ **3.** $\left\{-\dfrac{3}{2},2\right\}$ **5.** $\{-2,2\}$ **7.** $\{-8,8\}$ **9.** $\{-\sqrt{11},\sqrt{11}\}$ **11.** $\{2\sqrt{5},-2\sqrt{5}\}$ **13.** $\{-\sqrt{3},\sqrt{3}\}$ **15.** $\{-4\sqrt{2},4\sqrt{2}\}$ **17.** $\{-3,3\}$ **19.** $\{-\sqrt{6},\sqrt{6}\}$ **21.** $\{-5\sqrt{2},5\sqrt{2}\}$ **23.** $\{-2\sqrt{2},2\sqrt{2}\}$ **25.** $\{-2\sqrt{2},2\sqrt{2}\}$ **27.** $\{-\sqrt{2},\sqrt{2}\}$ **29.** $\left\{-\dfrac{\sqrt{6}}{5},\dfrac{\sqrt{6}}{5}\right\}$ **31.** $\{-\sqrt{11},\sqrt{11}\}$ **33.** $\{-4,0\}$ **35.** $\{-1,9\}$ **37.** $\{-3-\sqrt{6},-3+\sqrt{6}\}$ **39.** $\{9-3\sqrt{2},9+3\sqrt{2}\}$ **41.** $\{-5+4\sqrt{2},-5-4\sqrt{2}\}$ **43.** $\{-a-6,-a+6\}$ **45.** $\{6-a,6+a\}$ **47.** $\{p-q,p+q\}$ **49.** $\left\{-\dfrac{1}{2},\dfrac{7}{2}\right\}$ **51.** 5 meters **53.** 2 feet **55.** 9, -9 **57.** 0,9 **59.** 7 inches, 14 inches **61.** length = 24 meters; width = 6 meters **63.** 4 and 8 **65.** $5\sqrt{2}$ centimeters

Solutions to trial exercise problems

11. $a^2 = 20$
Extract the roots.
$$a = \sqrt{20} \text{ or } a = -\sqrt{20}$$
$$a = \sqrt{4\cdot 5} \text{ or } a = -\sqrt{4\cdot 5}$$
$$a = 2\sqrt{5} \text{ or } a = -2\sqrt{5}$$
$\{2\sqrt{5},-2\sqrt{5}\}$

18. $5x^2 = 75$
Divide each member by 5.
$$x^2 = 15$$
Extract the roots.
$$x = \sqrt{15} \text{ or } x = -\sqrt{15}$$
$\{\sqrt{15},-\sqrt{15}\}$

25. $\frac{3}{4}x^2 - 6 = 0$
Multiply each member by 4.
$3x^2 - 24 = 0$
Add 24 to each member.
$3x^2 = 24$
Divide each member by 3.
$x^2 = 8$
Then $x = \sqrt{8}$ or $x = -\sqrt{8}$
$x = 2\sqrt{2}$ or $x = -2\sqrt{2}$
$\{2\sqrt{2}, -2\sqrt{2}\}$

33. $(x + 2)^2 = 4$
Extract the roots.
$x + 2 = \pm 2$
Add -2 to each member.
$x = -2 \pm 2$
So $x = -2 + 2$ or $x = -2 - 2$
$x = 0$ or -4
$\{0, -4\}$

39. $(x - 9)^2 = 18$
Extract the roots.
$x - 9 = \pm\sqrt{18} = \pm 3\sqrt{2}$
Add 9 to each member.
$x = 9 + 3\sqrt{2}$ or $x = 9 - 3\sqrt{2}$
$\{9 + 3\sqrt{2}, 9 - 3\sqrt{2}\}$

44. $(x - a)^2 = 50$
$x - a = \sqrt{50} = 5\sqrt{2}$ or
$x - a = -\sqrt{50} = -5\sqrt{2}$
Add a to each member.
$x = a + 5\sqrt{2}$ or $x = a - 5\sqrt{2}$
$\{a + 5\sqrt{2}, a - 5\sqrt{2}\}$

58. Let n = the number. Then n^2 = the square of the number and $8n$ = eight times the number.
The equation is $2n^2 - 8n = 0$
$2n(n - 4) = 0$
$2n = 0$ or $n - 4 = 0$
$n = 0$ or $n = 4$
The number n is 0 or 4.

61. Using $A = \ell w$, let ℓ = length of the rectangle. Then $\frac{1}{4}\ell$ = width of the rectangle.
The equation is $\ell \cdot \frac{1}{4}\ell = 144$
$\frac{1}{4} \cdot \ell^2 = 144$
$\ell^2 = 576$
$\ell = \pm\sqrt{576} = \pm 24$
Since length cannot be negative, then $\ell = 24$ meters and $w = \frac{1}{4}(24) = 6$ meters.

Review exercises

1. $x^2 - 4x + 4$ 2. $9z^2 + 12z + 4$ 3. $(x + 9)^2$
4. $(3y + 5)^2$ 5. $\frac{3x^2 - 7x}{(x + 2)(x - 2)}$ 6. $\frac{1}{(x - 2)(x + 3)}$

Exercise 10–2

Answers to odd-numbered problems

1. $x^2 + 10x + 25$; $(x + 5)^2$ 3. $a^2 - 12a + 36$; $(a - 6)^2$
5. $x^2 + 24x + 144$; $(x + 12)^2$ 7. $y^2 - 20y + 100$; $(y - 10)^2$
9. $x^2 + x + \frac{1}{4}$; $\left(x + \frac{1}{2}\right)^2$ 11. $x^2 - 7x + \frac{49}{4}$; $\left(x - \frac{7}{2}\right)^2$
13. $x^2 + \frac{1}{2}x + \frac{1}{16}$; $\left(x + \frac{1}{4}\right)^2$ 15. $s^2 - \frac{1}{5}s + \frac{1}{100}$; $\left(s - \frac{1}{10}\right)^2$ 17. $y^2 + \frac{2}{3}y + \frac{1}{9}$; $\left(y + \frac{1}{3}\right)^2$
19. $m^2 - \frac{2}{5}m + \frac{1}{25}$; $\left(m - \frac{1}{5}\right)^2$ 21. $a^2 - \frac{3}{2}a + \frac{9}{16}$; $\left(a - \frac{3}{4}\right)^2$ 23. $\{-7, -1\}$ 25. $\{-2, 6\}$ 27. $\{1, 3\}$
29. $\left\{\frac{1 - \sqrt{5}}{2}, \frac{1 + \sqrt{5}}{2}\right\}$ 31. $\left\{\frac{5 - \sqrt{17}}{2}, \frac{5 + \sqrt{17}}{2}\right\}$
33. $\{2 - \sqrt{85}, 2 + \sqrt{85}\}$ 35. $\left\{\frac{-21 - \sqrt{401}}{2}, \frac{-21 + \sqrt{401}}{2}\right\}$
37. $\left\{-\frac{3}{2}, 1\right\}$ 39. $\left\{-3, -\frac{1}{2}\right\}$ 41. $\left\{-\frac{1}{2}, \frac{3}{2}\right\}$ 43. $\left\{\frac{2}{3}, \frac{3}{2}\right\}$
45. $\left\{\frac{-1 - \sqrt{13}}{2}, \frac{-1 + \sqrt{13}}{2}\right\}$ 47. $\left\{-\frac{2}{3}, \frac{1}{2}\right\}$
49. $\{3 - \sqrt{5}, 3 + \sqrt{5}\}$ 51. $\left\{\frac{1 - \sqrt{57}}{4}, \frac{1 + \sqrt{57}}{4}\right\}$
53. $\left\{\frac{-1 - \sqrt{29}}{2}, \frac{-1 + \sqrt{29}}{2}\right\}$ 55. $\left\{\frac{-5 - \sqrt{17}}{4}, \frac{-5 + \sqrt{17}}{4}\right\}$
57. 12 inches; 8 inches 59. $\ell = 15$ millimeters; $w = 7$ millimeters
61. 17 inches by 9 inches 63. $\frac{11}{2}$ meters by $\frac{7}{2}$ meters
65. 14 rods by 6 rods 67. $w = 3$ inches

Solutions to trial exercise problems

3. $a^2 - 12a$
Square one-half of the coefficient of a, -12.
$\left[\frac{1}{2}(-12)\right]^2 = (-6)^2 = 36$
Then $a^2 - 12a + 36 = (a - 6)^2$

9. $x^2 + x$
Square one-half of the coefficient of x, 1.
$\left[\frac{1}{2}(1)\right]^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$
So $x^2 + x + \frac{1}{4} = \left(x + \frac{1}{2}\right)^2$

13. $x^2 + \frac{1}{2}x$
Square one-half of the coefficient of x, $\frac{1}{2}$.
$\left[\frac{1}{2}\left(\frac{1}{2}\right)\right]^2 = \left(\frac{1}{4}\right)^2 = \frac{1}{16}$
So $x^2 + \frac{1}{2}x + \frac{1}{16} = \left(x + \frac{1}{4}\right)^2$

16. $x^2 - \frac{3}{8}x$
Square one-half of the coefficient of x, $-\frac{3}{8}$.
$\left[\frac{1}{2}\left(-\frac{3}{8}\right)\right]^2 = \left(-\frac{3}{16}\right)^2 = \frac{9}{256}$
So $x^2 - \frac{3}{8}x + \frac{9}{256} = \left(x - \frac{3}{16}\right)^2$

29. $u^2 - u - 1 = 0$
Add 1 to each member.
$u^2 - u = 1$
Add $\left[\frac{1}{2}(-1)\right]^2 = \left(-\frac{1}{2}\right)^2 = \frac{1}{4}$ to each member.
$u^2 - u + \frac{1}{4} = 1 + \frac{1}{4}$
Then $\left(u - \frac{1}{2}\right)^2 = \frac{5}{4}$ Then
$u - \frac{1}{2} = \pm\sqrt{\frac{5}{4}} = \pm\frac{\sqrt{5}}{2}$
so $u = \frac{1}{2} \pm \frac{\sqrt{5}}{2} = \frac{1 \pm \sqrt{5}}{2}$
Then $u = \frac{1 + \sqrt{5}}{2}$ or $u = \frac{1 - \sqrt{5}}{2}$
$\left\{\frac{1 + \sqrt{5}}{2}, \frac{1 - \sqrt{5}}{2}\right\}$

36. $3x^2 + 6x = 3$
Divide each term by 3.
$x^2 + 2x = 1$
Then $\left[\frac{1}{2}(2)\right]^2 = (1)^2 = 1$, so add 1 to each member.
$x^2 + 2x + 1 = 1 + 1$
So $(x + 1)^2 = 2$. Then
$x + 1 = \pm\sqrt{2}$
so $x = -1 \pm\sqrt{2}$
$x = -1 + \sqrt{2}$ or $x = -1 - \sqrt{2}$
$\{-1 + \sqrt{2}, -1 - \sqrt{2}\}$

37. $2x^2 + x - 3 = 0$
Add 3 to each member to get $2x^2 + x = 3$.
Now divide each term by 2.
$x^2 + \frac{1}{2}x = \frac{3}{2}$
Add $\left[\frac{1}{2}\left(\frac{1}{2}\right)\right]^2 = \left(\frac{1}{4}\right)^2 = \frac{1}{16}$ to each member. Then
$x^2 + \frac{1}{2}x + \frac{1}{16} = \frac{3}{2} + \frac{1}{16}$
$\left(x + \frac{1}{4}\right)^2 = \frac{25}{16}$
Extract the roots.
$x + \frac{1}{4} = \sqrt{\frac{25}{16}}$ or $x + \frac{1}{4} = -\sqrt{\frac{25}{16}}$
So $x = -\frac{1}{4} \pm \frac{5}{4}$ and we have
$x = -\frac{1}{4} + \frac{5}{4} = \frac{4}{4} = 1$ or $x = -\frac{1}{4} - \frac{5}{4} = \frac{-6}{4} = -\frac{3}{2}$.
$\left\{1, -\frac{3}{2}\right\}$

48. $4 - x^2 = 2x$
Add x^2 to each member.
$4 = x^2 + 2x$ or $x^2 + 2x = 4$
Then add $\left[\frac{1}{2}(2)\right]^2 = (1)^2 = 1$ to each member.
$x^2 + 2x + 1 = 4 + 1$
so $(x + 1)^2 = 5$
Extract the roots.
$x + 1 = \sqrt{5}$ or $x + 1 = -\sqrt{5}$
Then $x = -1 + \sqrt{5}$ or $x = -1 - \sqrt{5}$
$\{-1 + \sqrt{5}, -1 - \sqrt{5}\}$

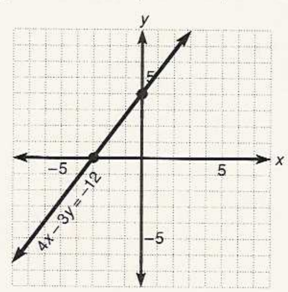
53. $(x + 3)(x - 2) = 1$
Perform the indicated multiplication in the left member.
$x^2 + x - 6 = 1$
Add 6 to each member to get $x^2 + x = 7$.
Add $\left[\frac{1}{2}(1)\right]^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$ to each member.
Thus $x^2 + x + \frac{1}{4} = 7 + \frac{1}{4}$
and $\left(x + \frac{1}{2}\right)^2 = \frac{29}{4}$
Then $x + \frac{1}{2} = \pm\sqrt{\frac{29}{4}} = \pm\sqrt{\frac{29}{2}}$
so $x = -\frac{1}{2} \pm \frac{\sqrt{29}}{2} = \frac{-1 \pm \sqrt{29}}{2}$
Then $x = \frac{-1 + \sqrt{29}}{2}$ or $x = \frac{-1 - \sqrt{29}}{2}$
$\left\{\frac{-1 + \sqrt{29}}{2}, \frac{-1 - \sqrt{29}}{2}\right\}$

59. By "a surface of a rectangular solid has a width w that is 8 millimeters shorter than its length ℓ," we get ℓ = the length of the part and
$\ell - 8$ = the width of the part. Then given area $A = 105$ square millimeters, and using $A = \ell w$, $\ell(\ell - 8) = 105$, then
$\ell^2 - 8\ell = 105$.
Add $\left[\frac{1}{2}(-8)\right]^2 = (-4)^2 = 16$ to both members.
$\ell^2 - 8\ell + 16 = 105 + 16$
$(\ell - 4)^2 = 121$
$\ell - 4 = \pm\sqrt{121} = \pm 11$
so $\ell - 4 = 11$ or $\ell - 4 = -11$
Then
$\ell = 4 + 11$ or $\ell = 4 - 11$
$\ell = 15$ $\quad\ell = -7$
Since a rectangle must have positive length, -7 is ruled out. So the length $\ell = 15$ millimeters and the width $\ell - 8 = 7$ millimeters.

Review exercises

1. $\sqrt{9} = 3$ **2.** $\sqrt{45} = 3\sqrt{5}$ **3.** $\{(2,0)\}$

4.



5. 9 dozen

Exercise 10–3

Answers to odd-numbered problems

1. $5x^2 - 3x + 8 = 0$; $a = 5$, $b = -3$, $c = 8$
3. $6z^2 + 2z - 1 = 0$; $a = 6$, $b = 2$, $c = -1$
5. $4x^2 - 2x + 1 = 0$; $a = 4$, $b = -2$, $c = 1$
7. $x^2 + 3x = 0$; $a = 1$, $b = 3$, $c = 0$
9. $5x^2 - 2 = 0$; $a = 5$, $b = 0$, $c = -2$
11. $p^2 + 3p - 4 = 0$; $a = 1$, $b = 3$, $c = -4$

13. $x^2 + 2x - 9 = 0; a = 1, b = 2, c = -9$
15. $8m^2 - 3m - 2 = 0; a = 8, b = -3, c = -2$ **17.** $\{1,2\}$
19. $\{1\}$ **21.** $\{-5,5\}$ **23.** $\{-\sqrt{2},\sqrt{2}\}$ **25.** $\{0,3\}$ **27.** $\left\{0,\frac{9}{5}\right\}$
29. $\left\{\frac{9+\sqrt{65}}{2},\frac{9-\sqrt{65}}{2}\right\}$ **31.** $\{-1-\sqrt{7}, -1+\sqrt{7}\}$
33. $\{4+\sqrt{15}, 4-\sqrt{15}\}$ **35.** $\left\{\frac{5-\sqrt{97}}{6},\frac{5+\sqrt{97}}{6}\right\}$
37. $\left\{\frac{-9-\sqrt{57}}{6},\frac{-9+\sqrt{57}}{6}\right\}$ **39.** $\left\{-\frac{5}{3},2\right\}$ **41.** $\{-4\}$
43. $\left\{\frac{5}{2}\right\}$ **45.** $\left\{-\frac{3}{2}\right\}$ **47.** $\left\{\frac{1-\sqrt{22}}{3},\frac{1+\sqrt{22}}{3}\right\}$
49. $\{-1-\sqrt{7},-1+\sqrt{7}\}$ **51.** $\left\{\frac{3-\sqrt{105}}{6},\frac{3+\sqrt{105}}{6}\right\}$
53. $\left\{\frac{5-\sqrt{85}}{10},\frac{5+\sqrt{85}}{10}\right\}$ **55.** $\left\{\frac{3-\sqrt{41}}{4},\frac{3+\sqrt{41}}{4}\right\}$
57. $\left\{-\frac{3}{2},3\right\}$ **59.** $\left\{\frac{-1-\sqrt{3}}{3},\frac{-1+\sqrt{3}}{3}\right\}$ **61. a.** 2 seconds
b. $\sqrt{6}$ seconds $\approx$ 2.5 sec **c.** $\frac{\sqrt{30}}{2}$ seconds $\approx$ 2.74 sec
63. a. $-20 + 20\sqrt{11}$ **b.** $\frac{-15+5\sqrt{329}}{2}$
65. $b = 9\sqrt{10}$ inches; $h = 3\sqrt{10}$ inches **67.** 6 millimeters; 8 millimeters **69.** $-1+\sqrt{7}; 1+\sqrt{7}$ **71.** $5\sqrt{19}$ feet
73. -16 and -14 **75.** 7 or $\frac{1}{7}$

Solutions to trial exercise problems

7. $x^2 = -3x$
Add $3x$ to each member. Then
$x^2 + 3x = 0$ and
$a = 1, b = 3, c = 0.$

12. $2x(x - 9) = 1$
Perform the indicated multiplication.
$2x^2 - 18x = 1$
Add -1 to each member.
$2x^2 - 18x - 1 = 0$, so
$a = 2, b = -18, c = -1.$

18. $y^2 + 6y + 9 = 0$
Here $a = 1$, $b = 6$, and $c = 9$ so
$$y = \frac{-6 \pm \sqrt{(6)^2 - 4(1)(9)}}{2(1)} = \frac{-6 \pm \sqrt{36-36}}{2} = \frac{-6 \pm \sqrt{0}}{2} = \frac{-6}{2} = -3$$
$\{-3\}$

21. $x^2 - 25 = 0$
We can write this
$x^2 + 0x - 25 = 0$, so
$a = 1, b = 0, c = -25.$
Then $x = \dfrac{-0 \pm \sqrt{0^2 - 4(1)(-25)}}{2(1)}$
$x = \dfrac{\pm\sqrt{100}}{2}$
so $x = \pm\dfrac{10}{2}$ Then
$x = 5$ or $x = -5$
$\{5,-5\}$

26. $x^2 = 4x$
Add $-4x$ to each member and write the equation as
$x^2 - 4x + 0 = 0$
So $a = 1, b = -4, c = 0,$
and $x = \dfrac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(0)}}{2(1)}$
$= \dfrac{4 \pm \sqrt{16}}{2}$
$= \dfrac{4 \pm 4}{2}$
Then $x = \dfrac{4+4}{2} = \dfrac{8}{2} = 4$ or $x = \dfrac{4-4}{2} = \dfrac{0}{2} = 0$
$\{0,4\}$

36. $4t^2 = 8t - 3$
Add $3 - 8t$ to each member.
$4t^2 - 8t + 3 = 0$
So $a = 4, b = -8, c = 3,$
and $t = \dfrac{-(-8) \pm \sqrt{(-8)^2 - 4(4)(3)}}{2(4)}$
$= \dfrac{8 \pm \sqrt{64-48}}{8}$
$= \dfrac{8 \pm \sqrt{16}}{8}$
so $t = \dfrac{8 \pm 4}{8}$
Then $t = \dfrac{8+4}{8} = \dfrac{12}{8} = \dfrac{3}{2}$ or $t = \dfrac{8-4}{8} = \dfrac{4}{8} = \dfrac{1}{2}$
$\left\{\frac{1}{2},\frac{3}{2}\right\}$

54. $2x^2 - \dfrac{7}{2} + \dfrac{x}{2} = 0$
Multiply by the LCD, 2.
$4x^2 - 7 + x = 0$
Then write in standard form.
$4x^2 + x - 7 = 0$
Then $a = 4$, $b = 1$, and $c = -7$.
Thus $x = \dfrac{-1 \pm \sqrt{1^2 - 4(4)(-7)}}{2(4)}$
$= \dfrac{-1 \pm \sqrt{1+112}}{8}$
$= \dfrac{-1 \pm \sqrt{113}}{8}$
So $x = \dfrac{-1+\sqrt{113}}{8}$ or $x = \dfrac{-1-\sqrt{113}}{8}$
$\left\{\frac{-1+\sqrt{113}}{8},\frac{-1-\sqrt{113}}{8}\right\}$

60. **a.** Using $s = vt + \frac{1}{2}at^2$, replace s with 8, v with 3, and a with 4.

$$8 = 3t + \frac{1}{2}(4)t^2$$
$$8 = 3t + 2t^2$$
$$2t^2 + 3t - 8 = 0$$
$$t = \frac{-3 \pm \sqrt{3^2 - 4(2)(-8)}}{2(2)}$$
$$= \frac{-3 \pm \sqrt{9 + 64}}{4}$$
$$= \frac{-3 \pm \sqrt{73}}{4}$$
$$t = \frac{-3 + \sqrt{73}}{4} \approx 1.39 \text{ or } t = \frac{-3 - \sqrt{93}}{4} \text{ (reject)}$$

66. Using $a^2 + b^2 = c^2$, replace a with x, b with $x + 14$, and c with $x + 16$.

$$x^2 + (x + 14)^2 = (x + 16)^2$$
$$x^2 + x^2 + 28x + 196 = x^2 + 32x + 256$$
$$2x^2 + 28x + 196 = x^2 + 32x + 256$$
$$x^2 - 4x - 60 = 0$$
$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-60)}}{2(1)}$$
$$= \frac{4 \pm \sqrt{16 + 240}}{2}$$
$$= \frac{4 \pm \sqrt{256}}{2}$$
$$= \frac{4 \pm 16}{2}$$

Then $x = \frac{4 + 16}{2} = \frac{20}{2} = 10$ or

$x = \frac{4 - 16}{2} = \frac{-12}{2} = -6$ (reject).

Thus, $x = 10$.

74. Let n = the first odd positive integer. Then $n + 2$ = the next consecutive odd positive integer.

The equation is then $n(n + 2) = 143$

$$n^2 + 2n = 143$$
$$n^2 + 2n - 143 = 0$$
$$(n + 13)(n - 11) = 0$$

Then $n = -13$ and $n + 2 = -11$ or $n = 11$ and $n + 2 = 13$. Reject -13 and -11 since we want positive integers. Thus 11 and 13 are two consecutive odd positive integers.

Review exercises

1. $3x^2 + x + 5$ **2.** $5y^2 + 33y - 14$ **3.** $16z^2 - 9$
4. $9x^2 - 30x + 25$ **5.** $\{-2,2\}$ **6.** $\left\{-\frac{1}{2},4\right\}$
7. $\left\{\frac{1 - \sqrt{41}}{2},\frac{1 + \sqrt{41}}{2}\right\}$ **8.** $\frac{2x - 8}{(x + 2)(x - 2)(x - 3)}$

Exercise 10–4

Answers to odd-numbered problems

1. $9 + 0i$ **3.** $0 + 4i$ **5.** $0 + 5i$ **7.** $4 + 4i$
9. $4 + i$ **11.** $1 - 4i$ **13.** $5 - 2i$ **15.** $-9 - 2i\sqrt{7}$
17. $-12 + 6i$ **19.** $10 + 11i$ **21.** 41 **23.** $-33 + 56i$
25. $\frac{15}{13} + \frac{10}{13}i$ **27.** $\frac{1}{17} + \frac{13}{17}i$ **29.** $\frac{17}{25} - \frac{19}{25}i$
31. $\{-2 + 4i, -2 - 4i\}$ **33.** $\left\{\frac{-1 + i\sqrt{7}}{2}, \frac{-1 - i\sqrt{7}}{2}\right\}$
35. $\left\{\frac{3 + i\sqrt{11}}{2},\frac{3 - i\sqrt{11}}{2}\right\}$ **37.** $\left\{\frac{-1 + i\sqrt{31}}{4},\frac{-1 - i\sqrt{31}}{4}\right\}$
39. $\left\{\frac{-1 + i\sqrt{19}}{2},\frac{-1 - i\sqrt{19}}{2}\right\}$ **41.** $b^2 - 4ac = 24$; two distinct irrational solutions **43.** $b^2 - 4ac = 0$; one rational solution **45.** $b^2 - 4ac = 9$; two distinct rational solutions
47. $b^2 - 4ac = 5$; two distinct irrational solutions

Solutions to trial exercise problems

7. $4 + 2\sqrt{-4} = 4 + 2(2i) = 4 + 4i$

14. $(1 - \sqrt{-4}) - (3 + \sqrt{-9})$
$= (1 - 2i) - (3 + 3i) = 1 - 2i - 3 - 3i$
$= (1 - 3) + (-2i - 3i)$
$= -2 + (-5i)$
$= -2 - 5i$

23. $(4 + 7i)^2$
$= 4^2 + 2(4)(7i) + (7i)^2 = 16 + 56i + 49i^2 = 16 + 56i + 49(-1)$
$= 16 + 56i - 49$
$= -33 + 56i$

28. $\frac{1 + i}{2 - i}\cdot\frac{2 + i}{2 + i} = \frac{(1 + i)(2 + i)}{2^2 + 1^2} = \frac{2 + 3i + i^2}{4 + 1} = \frac{2 + 3i + (-1)}{5}$
$= \frac{1 + 3i}{5} = \frac{1}{5} + \frac{3}{5}i$

38. $3y^2 - 2y + 3 = 0$. Here $a = 3$, $b = -2$, and $c = 3$.

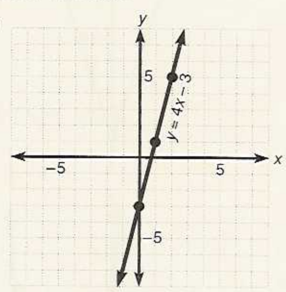
$$y = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(3)(3)}}{2(3)} = \frac{2 \pm \sqrt{4 - 36}}{6}$$
$$= \frac{2 \pm \sqrt{-32}}{6}$$
$$= \frac{2 \pm 4\sqrt{-2}}{6}$$
$$= \frac{2 \pm 4i\sqrt{2}}{6}$$
$$= \frac{2(1 \pm 2i\sqrt{2})}{6}$$
$$= \frac{1 \pm 2i\sqrt{2}}{3}$$

The solution set is $\left\{\frac{1 + 2i\sqrt{2}}{3},\frac{1 - 2i\sqrt{2}}{3}\right\}$.

47. $(x + 4)(x + 3) = 1$
$x^2 + 7x + 12 = 1$, and then we have $x^2 + 7x + 11 = 0$. Then $b^2 - 4ac = (7)^2 - 4(1)(11) = 49 - 44 = 5$; two distinct irrational solutions.

Review exercises

1.



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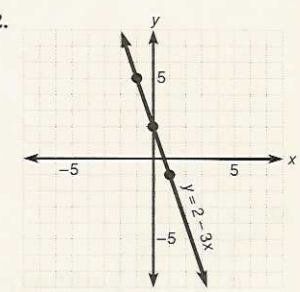
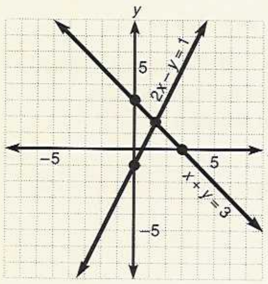


3. $\left(\frac{4}{3}, \frac{5}{3}\right)$



4. $8x - 3y = 17$ 5. $\frac{3x + 3}{4x}$

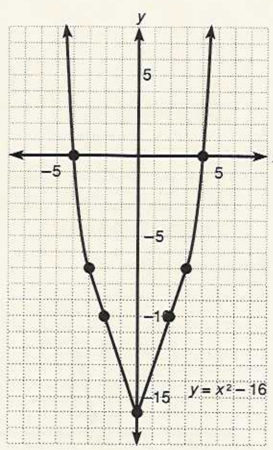
Exercise 10–5

Answers to odd-numbered problems

1. $(-1,-6)$, $(0,-4)$, $(3,14)$ **3.** $(-3,38)$, $(0,5)$, $(2,23)$ **5.** $(-6,198)$, $(0,0)$, $(6,162)$ **7.** $(-3,44)$, $(0,-1)$, $(5,124)$ **9.** $(-2,-4)$, $(0,4)$, $\left(\frac{3}{4}, \frac{13}{16}\right)$ **11.** y-intercept, -16; x-intercepts, 4 and -4 **13.** y-intercept, 8; x-intercepts, 4 and 2 **15.** y-intercept, 12; x-intercepts, -2 and -6 **17.** y-intercept, 5; x-intercepts, $\sqrt{5}$ and $-\sqrt{5}$ **19.** y-intercept, 9; x-intercept, -3 **21.** y-intercept, 5; x-intercepts, none **23.** y-intercept, 6; x-intercepts, none **25.** y-intercept, -16; x-intercept, 4 **27.** y-intercept, 1; x-intercepts, -1 and $-\frac{1}{2}$ **29.** y-intercept, 6; x-intercepts, -2 and $\frac{3}{2}$ **31.** $(0,-16)$; $x = 0$ **33.** $(3,-1)$; $x = 3$ **35.** $(-4,-4)$; $x = -4$ **37.** $(0,5)$; $x = 0$ **39.** $(-3,0)$; $x = -3$ **41.** $(0,5)$; $x = 0$ **43.** $(-2,2)$; $x = -2$ **45.** $(4,0)$; $x = 4$ **47.** $\left(-\frac{3}{4}, -\frac{1}{8}\right)$; $x = -\frac{3}{4}$ **49.** $\left(-\frac{1}{4}, \frac{49}{8}\right)$; $x = -\frac{1}{4}$

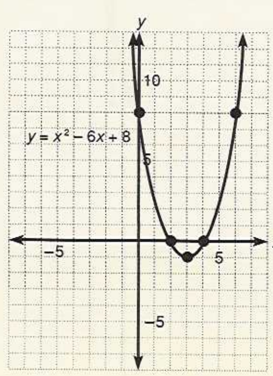
51.

x	y	
± 1	-15	arbitrary points
± 2	-12	
± 3	-7	
-4	0	x-intercepts
4	0	
0	-16	y-intercept; vertex



53.

x	y	
1	3	arbitrary points
5	3	
6	8	
3	-1	vertex
2	0	x-intercepts
4	0	
0	8	y-intercept



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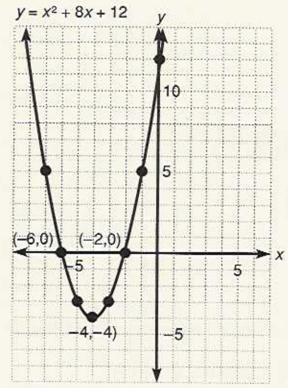

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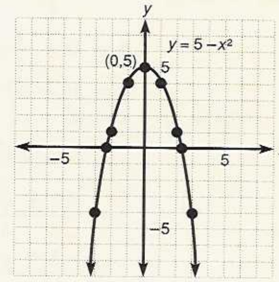

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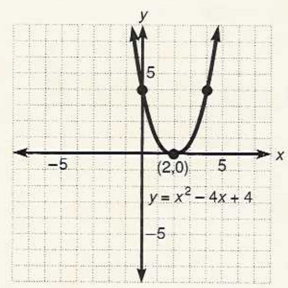

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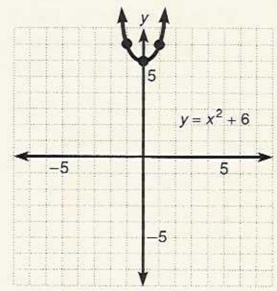

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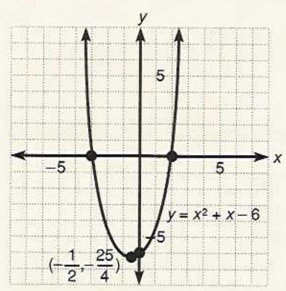

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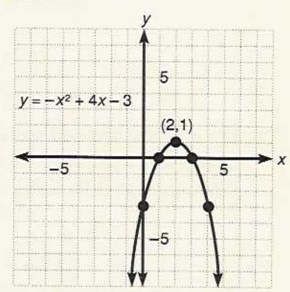

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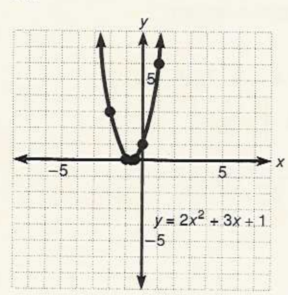

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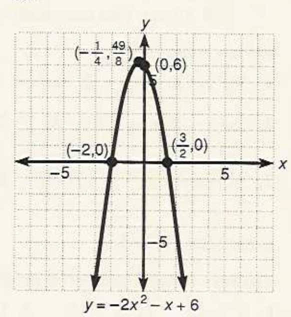

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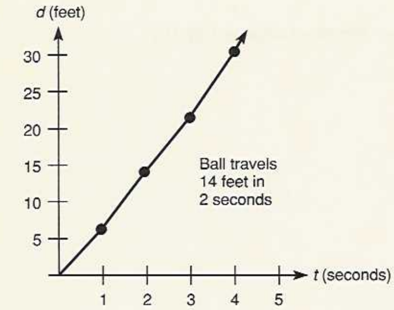

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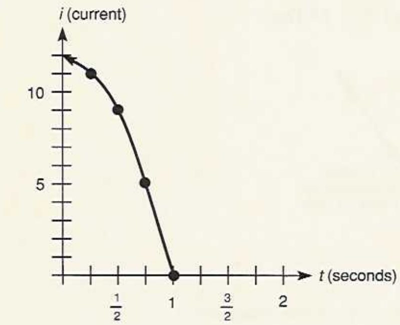

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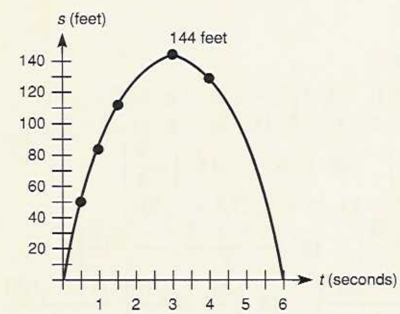

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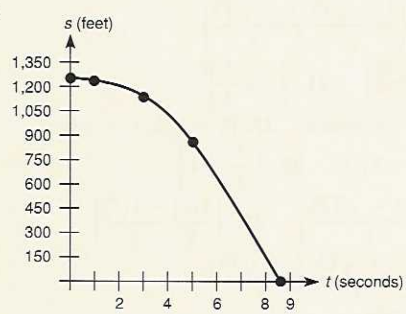


Solutions to trial exercise problems

3. $h(x) = 4x^2 + x + 5$

$h(-3) = 4(-3)^2 + (-3) + 5$
$= 4(9) - 3 + 5$
$= 36 - 3 + 5 = 38;\ (-3,38)$

$h(0) = 4(0)^2 + 0 + 5$
$= 0 + 0 + 5$
$= 5;\ (0,5)$

$h(2) = 4(2)^2 + 2 + 5$
$= 4(4) + 2 + 5$
$= 16 + 2 + 5$
$= 23;\ (2,23)$

13. $y = x^2 - 6x + 8$

Let $x = 0$, then $y = 0^2 - 6(0) + 8 = 8$.
Let $y = 0$, then $0 = x^2 - 6x + 8$. Factor the right member.
$0 = (x - 4)(x - 2)$
so $x = 4$ or $x = 2$
The y-intercept is 8 and the x-intercepts are 4 and 2.

17. $y = 5 - x^2$

Let $x = 0$, then $y = 5 - 0^2 = 5$.
Let $y = 0$, then $0 = 5 - x^2$. Add x^2 to each member.
$x^2 = 5$
Extract the roots.
$x = \sqrt{5}$ or $x = -\sqrt{5}$ so the y-intercept is 5 and the x-intercepts are $\sqrt{5}$and $-\sqrt{5}$.

27. $y = 2x^2 + 3x + 1$

Let $x = 0$, then $y = 2(0)^2 + 3(0) + 1 = 1$.
Let $y = 0$, then $0 = 2x^2 + 3x + 1$. Factor the right member.
$0 = (2x + 1)(x + 1)$
then $2x + 1 = 0$ or $x + 1 = 0$
so $x = -\dfrac{1}{2}$ or $x = -1$

The y-intercept is 1 and the x-intercepts are $-\dfrac{1}{2}$ and -1.

33. $y = x^2 - 6x + 8$

Here $a = 1$ and $b = -6$

so $x = \dfrac{-b}{2a} = -\dfrac{-6}{2(1)} = 3$

then $y = (3)^2 - 6(3) + 8$
$= 9 - 18 + 8$
$= -1$
The vertex is at $(3,-1)$. Axis of symmetry is $x = 3$.

37. $y = 5 - x^2$

Here $a = -1$ and $b = 0$

so $x = \dfrac{-b}{2a} = \dfrac{-0}{2(-1)} = 0$

then $y = 5 - 0^2 = 5$
Therefore the vertex is at $(0,5)$. Axis of symmetry is $x = 0$.

47. $y = 2x^2 + 3x + 1$

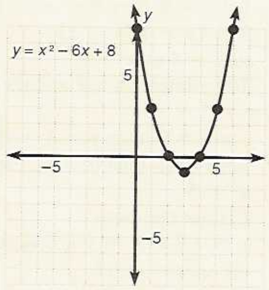
Here $a = 2$ and $b = 3$ so $x = \dfrac{-b}{2a} = \dfrac{-3}{2(2)} = -\dfrac{3}{4}$ Then

$$\begin{aligned} y &= 2\left(-\frac{3}{4}\right)^2 + 3\left(-\frac{3}{4}\right) + 1 \\ &= 2\left(\frac{9}{16}\right) - \frac{9}{4} + 1 \\ &= \frac{9}{8} - \frac{18}{8} + 1 \\ &= -\frac{9}{8} + 1 = -\frac{1}{8} \end{aligned}$$

The vertex is at $\left(-\frac{3}{4}, -\frac{1}{8}\right)$. Axis of symmetry is $x = -\frac{3}{4}$.

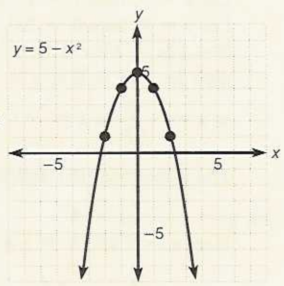
53. $y = x^2 - 6x + 8$

x	y	
0	8	y-intercept
4	0	x-intercepts
2	0	
3	-1	vertex
1	3	arbitrary points
5	3	
6	8	



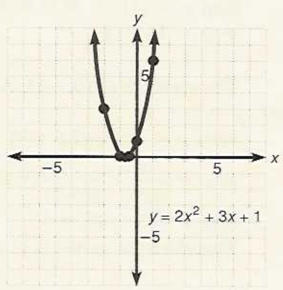
57. $y = 5 - x^2$

x	y	
0	5	y-intercept; vertex
$\sqrt{5}$	0	x-intercepts
$-\sqrt{5}$	0	
1	4	
2	1	arbitrary points
-2	1	
-1	4	



67. $y = 2x^2 + 3x + 1$

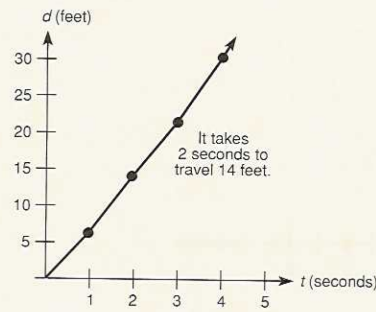
x	y	
0	1	y-intercept
$-\frac{1}{2}$	0	x-intercepts
-1	0	
$-\frac{3}{4}$	$-\frac{1}{8}$	vertex
1	6	arbitrary points
-2	3	
-3	10	



71. $d = 6t + \dfrac{t^2}{2}$ (*Note:* We must choose $t \geq 0$.)

t	d
0	0
1	$\frac{13}{2}$ or $6\frac{1}{2}$
2	14
3	$\frac{45}{2}$ or $22\frac{1}{2}$
4	32

$t = 2$ seconds when $d = 14$ feet



Chapter 10 review

1. $\{10, -10\}$ **2.** $\{-5, 5\}$ **3.** $\{\sqrt{2}, -\sqrt{2}\}$ **4.** $\{\sqrt{6}, -\sqrt{6}\}$
5. $\{2, -2\}$ **6.** $\{2\sqrt{3}, -2\sqrt{3}\}$ **7.** $\{4, -4\}$ **8.** $\{2\sqrt{3}, -2\sqrt{3}\}$
9. $\left\{\frac{3}{2}\sqrt{11}, -\frac{3}{2}\sqrt{11}\right\}$ **10.** $\{-6, 7\}$ **11.** $\left\{1, \frac{4}{3}\right\}$
12. $\{3 + \sqrt{5}, 3 - \sqrt{5}\}$ **13.** $\{5 + \sqrt{21}, 5 - \sqrt{21}\}$
14. $\left\{\frac{4 + \sqrt{14}}{2}, \frac{4 - \sqrt{14}}{2}\right\}$ **15.** $\left\{\frac{3 + 2\sqrt{6}}{3}, \frac{3 - 2\sqrt{6}}{3}\right\}$
16. $\left\{\frac{-5 + \sqrt{41}}{2}, \frac{-5 - \sqrt{41}}{2}\right\}$ **17.** $\left\{\frac{11 + \sqrt{101}}{2}, \frac{11 - \sqrt{101}}{2}\right\}$
18. $\left\{-\frac{3}{4}, 1\right\}$ **19.** $\left\{\frac{1 + \sqrt{13}}{2}, \frac{1 - \sqrt{13}}{2}\right\}$
20. $\left\{\frac{3 + \sqrt{41}}{8}, \frac{3 - \sqrt{41}}{8}\right\}$ **21.** $\left\{-2, \frac{5}{3}\right\}$
22. $w = 2$, meters, $\ell = 8$ meters **23.** $\{1 + \sqrt{6}, 1 - \sqrt{6}\}$
24. $\{-2 + 2\sqrt{3}, -2 - 2\sqrt{3}\}$ **25.** $\left\{\frac{5}{2}, -1\right\}$
26. $\left\{\frac{-7 + \sqrt{145}}{6}, \frac{-7 - \sqrt{145}}{6}\right\}$ **27.** $\left\{\frac{3\sqrt{2}}{2}, \frac{-3\sqrt{2}}{2}\right\}$
28. $\left\{0, -\frac{7}{4}\right\}$ **29.** $\left\{\frac{1 + \sqrt{13}}{3}, \frac{1 - \sqrt{13}}{3}\right\}$
30. $\left\{\frac{4 + \sqrt{34}}{6}, \frac{4 - \sqrt{34}}{6}\right\}$ **31.** 6 inches, 9 inches
32. $5 - i$ **33.** $4 + 9i$ **34.** $15 - 16i$ **35.** $3 + 21i$
36. 52 **37.** $16 - 30i$ **38.** $\frac{3}{2} + \frac{3}{2}i$ **39.** $\frac{4}{5} + \frac{8}{5}i$

40. $\frac{7}{10} - \frac{1}{10}i$ **41.** $\frac{26}{25} - \frac{7}{25}i$ **42.** $\{-2 + i\sqrt{3}, -2 - i\sqrt{3}\}$

43. $\left\{\frac{1 + i\sqrt{79}}{8}, \frac{1 - i\sqrt{79}}{8}\right\}$ **44.** $\{i\sqrt{7}, -i\sqrt{7}\}$

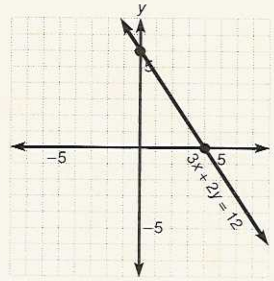
45. $\left\{\frac{-3 + 2i}{2}, \frac{-3 - 2i}{2}\right\}$ **46.** $b^2 - 4ac = 0$; one rational solution **47.** $b^2 - 4ac = 25$; two distinct rational solutions **48.** $b^2 - 4ac = -56$; two distinct complex solutions **49.** $b^2 - 4ac = 37$; two distinct irrational solutions **50.** $f(-5) = 35, f(0) = -5, f(1) = -7$ **51.** $g(-1) = -3, g(0) = 4, g(3) = 1$
52. $h(-3) = 30, h(0) = 0, h(4) = 72$
53. $f(-4) = -36, f(0) = 12, f(2) = 0$
54. y-intercept, -12; x-intercepts, $6, -2$; vertex, $(2, -16)$; $x = 2$
55. y-intercept, 1; x-intercepts, $1, \frac{1}{5}$; vertex, $\left(\frac{3}{5}, -\frac{4}{5}\right)$; $x = \frac{3}{5}$
56. y-intercept, 8; x-intercepts, $2, -4$; vertex, $(-1, 9)$; $x = -1$
57. y-intercept, 2; x-intercepts, $1, -\frac{2}{3}$; vertex, $\left(\frac{1}{6}, \frac{25}{12}\right)$; $x = \frac{1}{6}$
58. y-intercept, 0; x-intercepts, $0, \frac{2}{5}$; vertex, $\left(\frac{1}{5}, -\frac{1}{5}\right)$; $x = \frac{1}{5}$
59. y-intercept, 0; x-intercepts, $0, \frac{1}{3}$; vertex, $\left(\frac{1}{6}, \frac{1}{12}\right)$; $x = \frac{1}{6}$
60. y-intercept, -8; x-intercepts, $\sqrt{2}, -\sqrt{2}$; vertex, $(0, -8)$; $x = 0$
61. y-intercept, 9; x-intercepts, $3, -3$; vertex, $(0, 9)$; $x = 0$
62. y-intercept, 2; x-intercepts, none; vertex, $(0, 2)$; $x = 0$
63. y-intercept, 3; x-intercepts, none; vertex, $(-1, 2)$; $x = -1$
64. $4\frac{1}{2}$ sec **65.** $93\frac{1}{3}$ ft

Final examination

1. $>$ **2.** 42 **3.** 0 **4.** -52 **5.** x^6 **6.** $\frac{1}{2x^5}$ **7.** $-12x^3y^5$

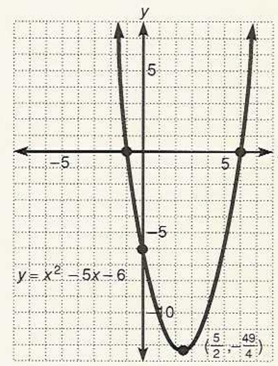
8. $\frac{y^6}{27x^3}$ **9.** 1 **10.** $7x^2 - 8y^2$ **11.** $-3y$ **12.** $y^2 - 81$
13. $49z^2 - 42zw + 9w^2$ **14.** $5x^3 + 17x^2 - 11x + 4$
15. $5y - 3x^3 + xy$ **16.** $4y + 1 + \frac{-2}{2y - 1}$ **17.** $\{7\}$
18. $\{12, -1\}$ **19.** $\left\{2, \frac{1}{3}\right\}$ **20.** $\left\{-\frac{8}{3}\right\}$ **21.** $\left\{0, \frac{3}{2}\right\}$
22. $3x(x - 2y + 3)$ **23.** $(a - 7)(a + 3)$
24. $(2x - 1)(2x - 5)$ **25.** $(3a + 8)(3a - 8)$
26. $(2a + b)(3x - y)$ **27.** $(x - 5)^2$ **28.** 11 and 12 or -12 and -11 **29.** $\frac{x + 1}{x + 2}$ **30.** $\frac{x - 2}{12}$ **31.** $\frac{14}{x - 6}$
32. $\frac{x^2 - 6x + 14}{(x + 5)(x - 5)}$ **33.** $\frac{5y + 4}{4y - 6}$ **34.** $x = \frac{16}{5}$ **35.** $\frac{21}{40}$ or 21:40
36. $2x + y = 1$ **37.** $m = \frac{2}{3}$; $b = -3$ **38.** $f(2) = 17, (2, 17)$; $f(0) = -1, (0, -1)$; $f(-1) = -1, (-1, -1)$ **39.** $\{(-19, -11)\}$
40. length = 12 feet; width = 5 feet
41. $-\sqrt{3}$ **42.** $\sqrt{6} + 3$ **43.** 13 **44.** $11 - 4\sqrt{7}$
45. $\frac{9 + 3\sqrt{5}}{4}$ **46.** -3 **47.** 8 **48.** $\{-1, 0\}$

49.



50.

x	y	
-1	0	x-intercepts
6	0	
0	-6	y-intercept
$\frac{5}{2}$	$-\frac{49}{4}$	vertex
1	-10	arbitrary points
2	-12	
3	-12	
4	-10	



51. $\left\{\frac{7 + \sqrt{97}}{8}, \frac{7 - \sqrt{97}}{8}\right\}$ **52.** $1 - 19i$ **53.** $17 + 9i$ **54.** 65

55. $-24 + 70i$ **56.** $10 + 2i$ **57.** 6 **58.** $\frac{27}{37} - \frac{23}{37}i$

59. $-\frac{6}{13} + \frac{15}{13}i$

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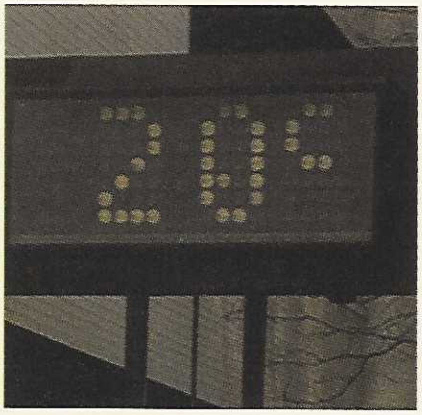

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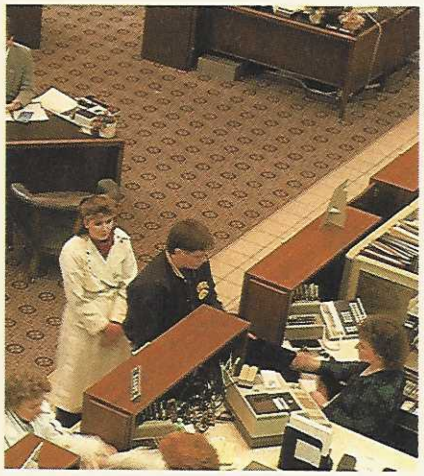

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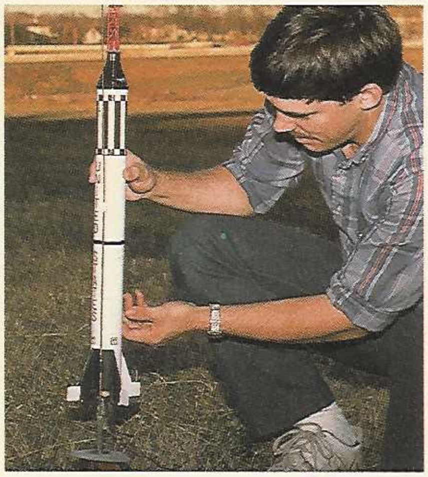

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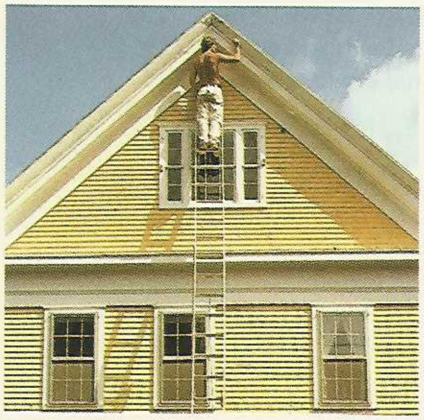

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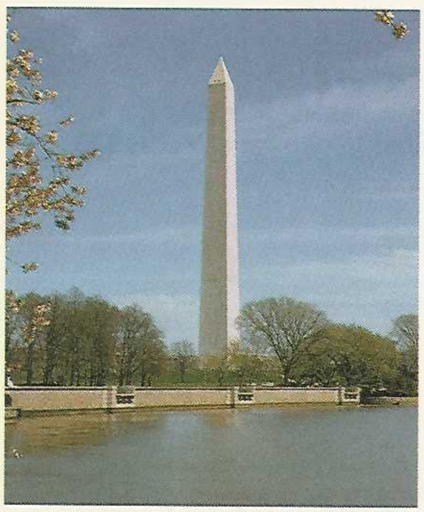

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